**Secondary Scheme of Work: Stage 10 Lite**

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| *Unit* | *Lessons* | *Key ‘Build a Mathematician’ (BAM) Indicators* | *Essential knowledge* |
| [Investigating properties of shapes](#IPS) | 12 | * [Solve problems involving direct and inverse proportion](http://kangaroomaths.com/free_resources/assessment/BAM/10M2_BAM.pdf)
* [Solve quadratic equations by factorising](http://kangaroomaths.com/free_resources/assessment/BAM/10M6_BAM.pdf)
* [Apply trigonometry in two dimensions](http://kangaroomaths.com/free_resources/assessment/BAM/10M10_BAM.pdf)
* [Calculate volumes of spheres, cones and pyramids](http://kangaroomaths.com/free_resources/assessment/BAM/10M11_BAM.pdf)
* [Understand and use vectors](http://kangaroomaths.com/free_resources/assessment/BAM/10M12_BAM.pdf)
 | * [Know the convention for labelling the sides in a right-angle triangle](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* [Know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* Know the exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°
* Know the exact value of tanθ for θ = 0°, 30°, 45° and 60°
* Know the information required to describe a transformation
* Know the special case of the difference of two squares
* Know set notation
* [Know the formulae for the volume of a sphere, a cone and a pyramid](http://www.kangaroomaths.com/free_resources/display/volumes.pdf)
* Know the formulae for the surface area of a sphere, and the curved surface area of a cone
* Know the meaning of roots, intercepts and turning points
 |
| [Calculating](#CALC) | 2 |
| [Solving equations and inequalities I](#SEI1) | 6 |
| [Mathematical movement I](#MM1) | 5 |
| [Algebraic proficiency: tinkering](#APT) | 4 |
| [Proportional reasoning](#PR) | 7 |
| [Pattern sniffing](#PS) | 2 |
| [Calculating space](#CS) | 9 |
| [Exploring fractions, decimals and percentages](#EFDP) | 5 |
| [Algebraic proficiency: visualising](#APV1) | 3 |
| [Solving equations and inequalities II](#SEI3) | 7 |
| [Analysing statistics](#AS) | 2 |
| [Mathematical movement II](#MM2) | 5 |
| Total: | 69 | [Stage 10 BAM Progress Tracker Sheet](http://www.kangaroomaths.com/free_resources/planning/stage10_tracker.pdf) |  |

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| **Maths Calendar** | *Based on 8 maths lessons per fortnight* |
| Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 |
| [Investigating properties of shapes](#IPS) | [Calc](#CALC) | [Solving equations I](#SEI1) | [Movement I](#MM1) | [Tinkering](#APT) | [Proportional reasoning](#PR) | [Pattern](#PS) | [Calculating space](#CS) | [Exploring FDP](#EFDP) |
| [10M10 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M10_BAM.pdf) |  |  |  |  | [10M2 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M2_BAM.pdf) |  | [10M11 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M11_BAM.pdf) |  |
| Week 14 | Week 15 | Week 16 | Week 17 | Week 18 | Week 19 | Week 20 | Week 21 | Week 22 | Week 23 | Week 24 | Week 25 | Week 26 |
| Assessment | [Visualising](#APV1) | [Solving equations II](#SEI3) | [Stats](#AS) | [Movement II](#MM2) | The Final Countdown |
|  |  | [10M6 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M6_BAM.pdf) |  | [10M12 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M12_BAM.pdf) |  |
| Week 27 | Week 28 | Week 29 | Week 30 | Week 31 | Week 32 | Week 33 | Week 34 | Week 35 | Week 36 | Week 37 | Week 38 | Week 39 |
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| *Investigating properties of shapes* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * make links to similarity (including trigonometric ratios) and scale factors
* know the exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tanθ for θ = 0°, 30°, 45° and 60°
* know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent
* apply it to find angles and lengths in right-angled triangles in two dimensional figures
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate similar triangles
* Explore trigonometry in right-angled triangles
* Set up and solve trigonometric equations
* Use trigonometry to solve practical problems

**Bring on the Maths: GCSE Higher Shape**Investigating angles: #5, #6, #7, #8, #9 | * Appreciate that the ratio of corresponding sides in similar triangles is constant
* Choose an appropriate trigonometric ratio that can be used in a given situation
* Understand that sine, cosine and tangent are functions of an angle
* Establish the exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°
* Establish the exact value of tanθ for θ = 0°, 30°, 45° and 60°
* Use a calculator to find the sine, cosine and tangent of an angle
* Know the trigonometric ratios, sinθ = opp/hyp, cosθ = adj/hyp, tanθ = opp/adj
* Set up and solve a trigonometric equation to find a missing side in a right-angled triangle
* Set up and solve a trigonometric equation when the unknown is in the denominator of a fraction
* Set up and solve a trigonometric equation to find a missing angle in a right-angled triangle
* Use trigonometry to solve problems involving bearings
* Use trigonometry to solve problems involving an angle of depression or an angle of elevation
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand and work with similar shapes
* Solve linear equations, including those with the unknown in the denominator of a fraction
* Understand and use Pythagoras’ theorem
 | SimilarOppositeAdjacentHypotenuseTrigonometryFunctionRatioSineCosineTangentAngle of elevation, angle of depression**Notation**sinθ stands for the ‘sine of θ’sin-1 is the inverse sine function, and not 1÷ sin | Ensure that all students are aware of the importance of their scientific calculator being in degrees mode.Ensure that students do not round until the end of a multi-step calculationThis unit of trigonometry should focus only on right-angled triangles in two dimensions. The sine rule, cosine rule, and applications in three dimensions are covered in Stage 11.NRICH: [History of Trigonometry](http://nrich.maths.org/6843)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students explore sets of similar triangles with angles of (at least) 30°, 45° and 60° as an introduction to the three trigonometric ratios**The mnemonic ‘Some Of Harry’s Cats Are Heavier Than Other Animals’ is used to help students remember the trigonometric ratios* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an angle and its exact sine (cosine / tangent). And another …
* Convince me that you have chosen the correct trigonometric function
* (When exploring sets of similar triangles and working out ratios in corresponding cases) why do you think that the results are all similar, but not the same? Could we do anything differently to get results that are closer? How could we make a final conclusion for each ratio?
 | KM: [From set squares to trigonometry](http://kangaroomaths.com/free_resources/teaching/geometry/set_squares_trigonometry.docx)KM: [Trigonometry flowchart](http://kangaroomaths.com/free_resources/ks4/resources/trig_flowchart.doc)NRICH: [Trigonometric protractor](http://nrich.maths.org/5601)NRICH: [Sine and cosine](http://nrich.maths.org/5671)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M10 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/10M10_BAM.pdf) | * Some students may not appreciate the fact that adjacent and opposite labels are not fixed, and are only relevant to a particular acute angle. In situations where both angles are given this can cause difficulties.
* Some students may not balance an equation such as sin35 = 4/x correctly, believing that the next step is (sin35)/4 = x
* Some students may think that sin-1θ = 1 ÷ sinθ
* Some students may think that sinθ means sin × θ
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| *Calculating* | *2 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * calculate with roots, and with integer indices
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Calculate with powers and roots
 | * Know and use the fact that a-n = 1/an
* Use the functionality of a scientific calculator when calculating with roots and powers
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate with positive indices using written methods and negative indices in the context of standard form
* Know the multiplication and division laws of indices
 | Power, RootIndex, IndicesStandard form**Notation** | NCETM: [Departmental workshops: Index Numbers](https://www.ncetm.org.uk/resources/13249)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pattern sniffing is encouraged to establish the result a0 = 1, a-n = 1/an , i.e.**23 = 2 × 2 × 2 = 8, 22= 2 × 2 = 4, 21= 2, 20= 1, 2-1=* $\frac{1}{2}$ |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that 2-3 = $\frac{1}{8}$
 | KM: [Maths to Infinity: Standard form](http://kangaroomaths.com/free_resources/infinity/standard_form.xlsm)KM: [Maths to Infinity: Indices](http://kangaroomaths.com/free_resources/infinity/indices.xlsm)NRICH: [Powers and Roots – Short Problems](http://nrich.maths.org/9324)NRICH: [Power Countdown](http://nrich.maths.org/6448)[Powers of 10](http://www.eamesoffice.com/the-work/powers-of-ten/) (external website)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may think that negative indices change the sign of a number, for example 2-1= -2 rather than 2-1= $\frac{1}{2}$
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| *Solving equations and inequalities I* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * solve two linear simultaneous equations in two variables algebraically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve simultaneous equations
* Solve problems involving simultaneous equations
 | * Understand the concept of solving simultaneous equations by substitution
* Decide whether to use elimination or substitution to solve a pair of simultaneous equations
* Solve two linear simultaneous equations in two variables by substitution
* Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)
* Derive and solve two simultaneous equations in complex cases
* Interpret the solution to a pair of simultaneous equations
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of solving simultaneous equations by elimination
* Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
* Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)
 | UnknownSolveSimultaneous equationsSubstitutionElimination **Notation** | Pupils have been introduced to solving simultaneous equations using elimination in simple cases in Stage 9. This includes either no multiplication being required or multiplication of just one equation being required. Solving simultaneous equations using substitution is new to this Stage. NCETM: [Departmental workshops: Simultaneous equations](https://www.ncetm.org.uk/resources/10340)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils are taught to label the equations (1) and (2), and label the subsequent equations (3), (4), etc.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of simultaneous equations with a solution x = 4, y = -2. And another. And another …
* Convince me x + 2y = 11, 3x + 4y = 18 can be solved using substitution and using elimination. Which method is best in this case?
* Always/ Sometimes/ Never: Solving a pair of simultaneous equations using elimination is more efficient than using substitution
 | KM: [Stick on the Maths: ALG2 Simultaneous linear equations](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7alg2_ewb.doc)KM: [Convinced?: ALG2 Simultaneous linear equations](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7alg2.pdf)NRICH: [Matchless](http://nrich.maths.org/5674)AQA: [Bridging Units Resource Pocket 4](http://allaboutmaths.aqa.org.uk/attachments/5309.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may not check the solution to a pair of simultaneous equations satisfy both equations
* Some pupils may not multiply all coefficients, or the constant, when multiplying an equation
* Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations
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| *Mathematical movement I* | *5 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including fractional scale factors)
* make links *between* similarity and scale factors
* apply the concepts of congruence and similarity, including the relationships between length in similar figures
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore enlargement of 2D shapes
* Investigate the transformation of 2D shapes
 | * Use the centre and scale factor to carry out an enlargement of a 2D shape with a fractional scale factor
* Find the scale factor and centre of an enlargement with fractional scale factor
* Perform a sequence of transformations on a 2D shape
* Find and describe a single transformation given two congruent 2D shapes
* Solve practical problems involving lengths in similar figures
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use the centre and scale factor to carry out an enlargement of a 2D shape with a positive integer scale factor
* Use the concept of scaling in diagrams
* Carry out reflection, rotations and translations of 2D shapes
* Find the scale factor of a given enlargement
 | Congruent, congruenceSimilarity, similar shapes, similar figuresEnlarge, enlargementScale factorTransformationRotationReflectionTranslation | Pupils have identified, described and constructed congruent shapes using rotation, reflection and translation in Stage 7. They have also identified, described and constructed similar shapes using enlargement in Stage 8 and experienced enlarging shapes using positive integer scale factors in Stage 9. Pupils also explore congruence and similarity - the use of proportion tables can be helpful to find the multiplier when solving similarity problems such as:

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|  | Shape A | Shape B |
| Known lengths | 6 | 9 |
| Missing lengths | 10 | 15 |
|  | **→ × 1.5 →** |

NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils should experience using dynamic software (e.g. Autograph) to explore enlargements using fractional scale factors* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of similar shapes. And another. And another …
* Always/ Sometimes/ Never: The resulting image of an enlargement is larger than the original object
* Kenny thinks rotating an object 90° about the origin followed by a reflection in the y-axis has the same effect as reflecting an object in the y-axis followed by a rotation 90° about the origin. Do you agree with Kenny? Explain your answer.
 | KM: [Enlargement 2](http://kangaroomaths.com/free_resources/teaching/geometry/enlargement_2.docx)KM: [Stick on the Maths SSM3: Enlargement (fractional scale factor)](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7ssm3.pdf)KM: [Stick on the Maths SSM1: Congruence and similarity](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8ssm1.pdf)NRICH: [Growing Rectangles](http://nrich.maths.org/6923)OCR: [Congruence Check In](http://www.ocr.org.uk/Images/222124-topic-check-in-9.02-congruence.pdf) and [Similarity Check In](http://www.ocr.org.uk/Images/222125-topic-check-in-9.04-similarity.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that the resulting image of an enlargement has to be larger than the original object.
* Some pupils may think that the order of transforming an object does not have an effect on the size and position of the final image.
* Some pupils may link scale factors and similarity using an additive, rather than multiplicative, relationship.
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| *Algebraic proficiency: tinkering* | *4 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * simplify and manipulate algebraic expressions by factorising quadratic expressions of the form x² + *b*x + *c*, including the difference of two squares
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Manipulate algebraic expressions
* Change the subject of a formula
 | * Use visual representations connected to the expanding of two binomials
* Identify when it is necessary to find two linear expressions to factorise a quadratic expression
* Factorise an expression involving the difference of two squares
* Change the subject of a formula when more than two steps are required
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate with negative numbers
* Multiply two linear expressions of the form (x ± a)(x ± b)
* Factorise a quadratic expression of the form x² + bx + c
 | EquivalentEquationExpressionExpandLinearQuadraticDifference of two squaresBinomialFactorise | Teachers also need to help pupils ‘see’ the difference of two squares by using pictorial representations NCETM: [Algebra](https://www.ncetm.org.uk/resources/43649)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Students are taught to use the grid method in reverse to factorise a quadratic**Students manipulate algebra tiles to explore factoring quadratics**The difference of two squares is explained using visual representation* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * The answer is 2x² + 10x + c. Show me a possible question. And another. And another …
* Convince me that 1032 – 972 = 1200 without a calculator.
* Convince me that 4x2 – 9 ≡ (3x – 2)(3x + 2).
* Jenny thinks that (3x – 2)2 = 3x2 + 12x + 4. Do you agree with Jenny? Explain your answer.
 | KM: [Maths to Infinity: Brackets](http://kangaroomaths.com/free_resources/infinity/brackets.xlsm)KM: [Maths to Infinity: Quadratics](http://kangaroomaths.com/free_resources/infinity/quadratics.xlsm)KM: [Stick on the Maths: Quadratic sequences](http://kangaroomaths.com/free_resources/teaching/sotm/level7/7alg5_ewb.doc)NRICH: [What’s possible?](http://nrich.maths.org/742)NRICH: [Finding Factors](http://nrich.maths.org/7452)[Algebra Tiles](http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles/AlgebraTiles.html) (external site)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Once pupils know how to factorise a quadratic expression of the form x² + bx + c they might overcomplicate the simpler case of factorising an expression such as 3x2 + 6x (≡ (3x + 0)(x + 2))
* Some pupils may think that (x + a)2 ≡ x2 + a2
* Some pupils may apply the ‘rules of factorising’ quadratics of the form x² + bx + c to quadratics of the form ax² + bx + c; e.g. 2x2 + 7x + 10 ≡ (2x + 5)(x + 2) because 2 × 5 = 10 and 2 + 5 = 7.
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| *Proportional reasoning* | *7 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Ratio and Proportion progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_RatioProportion.xlsx) |
| * interpret equations that describe direct and inverse proportion
* recognise and interpret graphs that illustrate direct and inverse proportion
* understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore differences between direct and inverse proportion
* Investigate ways of representing proportion in situation
* Solve problems involving proportion
 | * Recognise and interpret graph that illustrates direct proportion
* Recognise and interpret graph that illustrates inverse proportion
* Understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y
* Interpret equations that describe direct proportion
* Interpret equations that describe inverse proportion
* Solve problems which include finding the multiplier in a situation involving direct proportion
* Solve problems which include finding the multiplier in a situation involving inverse proportion
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the difference between direct and inverse proportion
* Recognise direct or inverse proportion in a situation
* Know the features of a graph that represents a direct or inverse proportion situation
* Know the features of an expression (or formula) that represents a direct or inverse proportion situation
* Understand the connection between the multiplier, the expression and the graph
 | Direct proportionInverse proportionMultiplier**Notation**$∝$ - ‘proportional to’ | Pupils have solved simple problems involving direct and inverse proportion in Stage 9. This unit focuses on developing a formal algebraic approach, including the use of proportionality constants, to solve direct and inverse proportion problems.NCETM: [Departmental workshops: Proportional Reasoning](https://www.ncetm.org.uk/resources/10334)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are taught to find a proportionality constant when solving problems* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an example of two quantities that will be in direct proportion. And another. And another …
* Convince me that this information shows a proportional relationship. What type of proportion is it?

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| 40 | 50 |
| 60 | 75 |
| 80 | 100 |

* Always/Sometimes/Never: X is inversely proportional to Y is equivalent to X is proportional to 1/Y
 | KM: [Graphing proportion](http://kangaroomaths.com/free_resources/teaching/number/graphing_proportion.docx)KM: [Investigating proportionality 2](http://kangaroomaths.com/free_resources/teaching/number/investigating_proportionality_2.docx)KM: [Stick on the Maths NNS1: Understanding Proportionality](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7nns1_ewb.doc)KM: [Stick on the Maths CALC1: Proportional Change and multiplicative methods](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7calc1_ewb.doc)KM: [Convinced: NNS1: Understanding Proportionality](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7nns1.pdf)KM: [Convinced: CALC1: Proportional Change and multiplicative methods](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7calc1.pdf)Hwb: [Inverse or direct?](http://hwb.wales.gov.uk/Resources/resource/6cc89f7e-a5c0-49fd-8a8d-909f38579549/en)NRICH: [In Proportion](http://nrich.maths.org/9268)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M2 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/10M2_BAM.pdf)  | * Some pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
* Some pupils may interpret *‘x is inversely proportional to y’* as y=x/k rather than y = k/x
* Some pupils may think that the proportionality constant always has to be greater than 1
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| *Pattern sniffing* | *2 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * recognise and use simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 )
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate geometric progressions
 | * Recognise and describe a simple geometric progression
* Find the next three terms, or any given term, in a geometric progression
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Find the nth term for an increasing linear sequence
* Find the nth term for an decreasing linear sequence
 | Termnth termGenerateFirst (second) differenceGeometric Progression**Notation**T(n) is often used to indicate the ‘nth term’ | In Stage 9, pupils recognised and used quadratic sequences. The focus in this stage is introducing pupils to geometric sequences (r>0).NCETM: [Departmental workshops: Sequences](https://www.ncetm.org.uk/resources/10337)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students should use a spreadsheet to explore aspects of sequences during this unit. For example, this could be using formulae to continue a given sequence, to generate the first few terms of a sequence from an nth term as entered, or to find the missing terms in sequence.**Ask pupils to repeatedly fold a piece of paper in half as many times as possible as an introduction to geometric sequences.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a geometric progression. And another. And another….
* Kenny thinks 1, 1, 1, 1, 1, … is an arithmetic sequence. Jenny thinks 1, 1, 1, 1, 1, … is a geometric sequence. Who is correct? Explain your answer.
 | KM: [Sequence plotting](http://kangaroomaths.com/free_resources/teaching/algebra/sequence_plotting.docx). A grid for plotting nth term against term.KM: [Maths to Infinity: Sequences](http://kangaroomaths.com/free_resources/infinity/sequences.xlsm) Hwb: [Linear and quadratic sequences](http://hwb.wales.gov.uk/Resources/resource/7548fc9f-26de-4f08-bd57-d6c1f051d879/en)NRICH: [Growing Surprises](http://nrich.maths.org/11212)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may think that it is possible to find an nth term for any sequence.
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| *Calculating space* | *9 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * calculatesurface area and volume of spheres, pyramids, cones and composite solids
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Calculate surface areas of solids
* Calculate volumes of solids
 | * Find the surface area of spheres
* Find the volume of spheres
* Use Pythagoras’ theorem to find lengths in a pyramid or cone
* Find the surface area of cones and pyramids
* Find the volume of cones and pyramids
* Identify how to find the volume of a composite solid
* Identify how to find the surface area of a composite solid
* Solve practical problems involving the surface area of solids
* Solve practical problems involving the volume of solids
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate exactly with multiples of π
* Know and use the formula for area and circumference of a circle
* Know how to use formulae to find the area of rectangles, parallelograms, triangles, trapezia, circles, sectors and
* Know how to find the area of compound shapes
* Know how to find the surface area of a right prism and a cylinder
* Calculate the surface area of a right prism and a cylinder
* Use Pythagoras’ theorem to find missing lengths in right-angled triangles
 | (Composite) solidSphere, Pyramid, ConePerpendicular (height), (slant height)Surface areaVolume**Notation**πAbbreviations of units in the metric system: km, m, cm, mm, mm2, cm2, m2, km2, mm3, cm3, km3 | Pupils have previously learnt how to find the surface area of right prisms and cylinders in Stage 9. This unit focuses on finding the volume and surface areas of cones, spheres and pyramids.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils explore the surface area of spheres using oranges (*[*https://www.youtube.com/watch?v=cAxHYFRx1Fs*](https://www.youtube.com/watch?v=cAxHYFRx1Fs) *)**Pupils explore volumes of pyramids by making nets of pyramids and prisms with the same polygonal base and using sand or sugar to compare volumes.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always/ Sometimes/ Never: The value of the volume of a pyramid is less than the value of the surface area of a pyramid.
* Always/ Sometimes/ Never: The value of the volume of a sphere is less than the value of the surface area of a sphere.
* Convince me that the volume of a pyramid = 1/3 × A × h
* Convince me that 1 m3 = 1 000 000 cm3
 | NRICH: [Surface Area and Volume](http://nrich.maths.org/9740) Hwb: [Summerhouse](http://hwb.wales.gov.uk/Resources/resource/dcc01b2d-ba39-449d-b5a1-a1c13de4c881/en) and [Radiators](http://hwb.wales.gov.uk/Resources/resource/305a31cd-8cec-4f4f-a507-f23b84c7b851/en)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M11 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/10M11_BAM.pdf) | * Some pupils will work out 4/3 × (π × r)3 when finding the volume of a sphere.
* Some pupils may confuse the concepts of surface area and volume
* Some pupils will work out 4 × (π × r)2 when finding the surface area of a sphere.
* Some pupils may think the volume of a pyramid = ½ × A × h
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| *Exploring fractions, decimals and percentages* | *5 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * set up, solve and interpret the answers in growth and decay problems, including compound interest
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve problems involving repeated percentage change
* Solve problems involving exponential growth and decay
 | * Recognise when a situation involves compound interest
* Set up a compound interest problem
* Calculate the result of a repeated percentage change, including compound interest
* Set up a growth or decay problem
* Solve problems involving growth and decay
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Move freely between terminating fractions, decimals and percentages
* Use a multiplier to calculate the result of percentage changes
 | FractionMixed numberTop-heavy fractionPercentage change, percentage increase, percentage increaseCompound interest, Simple interest (Exponential) growth, decay**Notation** | NRICH: [Teaching fractions with understanding](http://nrich.maths.org/2550)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches** |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Kenny thinks that the interest gained when £100 is increased 20% per annum for 4 years can be calculated by multiplying £100 by 2.0736. Do you agree with Kenny? Explain your answer.
 | KM: [Stick on the Maths 8: Repeated Proportional Change](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8calc1_ewb.doc)KM: [Convinced?: Repeated Proportional Change](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8calc1.pdf)NRICH: [Repetitiously](http://nrich.maths.org/1853/index) Hwb: [Borrowing money: APR](http://hwb.wales.gov.uk/Resources/resource/b0571745-fa5f-4303-a5c6-6e94369bce71/en), [Too good to be true!](http://hwb.wales.gov.uk/Resources/resource/e12648d5-ea68-4f26-bbf7-a67db5cb723e/en), [Double your money!](http://hwb.wales.gov.uk/Resources/resource/9d0ae110-d140-4f3c-b8a5-6f35b1917156/en) and [Comparing interest](http://hwb.wales.gov.uk/Resources/resource/225ba756-a1c7-4478-bc8c-e68631fc2b2b/en)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that an the amount created by increasing a quantity by 5% repeated four times is the same as increasing the quantity by 5% and multiplying that amount by 4.
* Some pupils may think the percentage multiplier for a 20% increase (or decrease) is 0.2
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| *Algebraic proficiency: visualising* | *3 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * identify and interpret roots, intercepts, turning points of quadratic functions graphically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate features of quadratic graphs
 | * Identify and interpret roots of quadratic functions graphically
* Identify and interpret intercepts of quadratic functions graphically
* Identify and interpret turning points of quadratic functions graphically
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Plot graphs of linear, quadratic, cubic and reciprocal functions
* Plot and interpret graphs of kinematic problems involving distance and speed
 | Function, equationLinear, non-linearQuadratic, cubic, reciprocalParabola, AsymptoteGradient, y-intercept, x-intercept, root**Notation***y* = *mx* + *c* | NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils use dynamic graphing software, e.g. Autograph and the ‘gradient value/function’, to explore graphs**All teachers explain the term ‘exponent’ to help students understand why ‘exponential’ functions are called ‘exponential’* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
|  | NRICH: [What’s that graph?](http://nrich.maths.org/7502)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think the graphs of all quadratic functions intercept the x-axis in one or two places.
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| *Solving equations and inequalities II* | *7 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * solve quadratic equations algebraically by factorising
* find approximate solutions to quadratic equations using a graph
* deduce roots of quadratic functions algebraically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve quadratic equations
* Use graphs to solve equations
 | * Solve a quadratic equation in factorised form
* Solve a quadratic equation of the form *x*² + *bx* + *c* by factorising
* Make connections between graphs and quadratic equations of the form *ax*² + *bx* + *c* = 0
* Make connections between graphs and quadratic equations of the form *ax*² + *bx* + *c* = *d*
* Find approximate solutions to quadratic equations using a graph
* Deduce roots of quadratic functions algebraically
* Solve problems that involve solving a quadratic equation in context
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Manipulate linear equations
* Factorise a quadratic expression of the form *x*² + *bx* + *c*
* Factorise a quadratic expression of the form *ax*² + *bx* + *c*
* Make connections between a linear equation and a graph
 | (Quadratic) equationFactoriseVariableUnknownManipulateSolveDeducex-interceptRoot | Pupils factorise quadratic expressions of the form ax2 + bx + c in Stage 9 (a = 1) and Stage 10. If A × B = 0 then either A = 0 or B = 0 is a fundamental underlying concept to solving quadratic equations when b ≠ 0 and c ≠ 0 by factorising.Pupils should experience solving quadratics with b ≠ 0 and c = 0, such as x2 + 6x = 0, and quadratics with b ≠ 0 and c ≠ 0, such as x2 + 6x + 8 = 0. Pupils may wish to ‘divide both sides by ‘x’ when solving quadratics such as x2 + 6x = 0 without appreciating that x could equal zero.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils are taught how to solve quadratics of the form ax*² + *bx* + *c* = 0 when: * b = 0 , b ≠ 0 and c = 0, b ≠ 0 and c ≠ 0

*Pupils are encouraged, whenever possible, to divide a quadratic equation by a common factor to make the factorising process easier, such as 2x2 + 6x + 8 = 0* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a quadratic equation with one solution x = 3. And another, and another …
* Always/Sometimes/Never: A quadratic equation can be solved by factorising.
* Convince me why you can’t ‘*divide both sides by x’* when solving x2 + 8x = 0
* Kenny is solving x2 + 6x + 8 = 2 as follows:

*(x + 4)(x + 2) = 2 so x + 4 = 2 or x + 2 = 1.**Therefore, x = -2 and x = -1.* * Do you agree with Kenny? Explain your answer.
 | NRICH: [How old am I?](http://nrich.maths.org/631)NRICH: [Golden thoughts](http://nrich.maths.org/271)Hwb: [Algebra Fails](https://hwb-live-storage.s3-eu-west-1.amazonaws.com/07/03/6e/1f/bbd642e183ffbaceb028194d/Q49_Algebra_fails_Resource.pdf?AWSAccessKeyId=AKIAJLXRMQJHU5RYD7ZQ&Expires=1448919073&Signature=OmY9fEVkeSzvX3HfqWH1ZeYmnGQ%3d)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M6 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/10M6_BAM.pdf) | * Some pupils may not appreciate that a quadratic equation must equal zero when solving by factorising
* Some pupils may solve x2 + 8x = 0 by dividing both sides by x to get x + 8 = 0, x = -8.
* Some pupils may forget to divide by the coefficient of x when solving quadratics such as 2x2 + 5x + 2 = 0, i.e. (2x + 1)(x + 2) = 0 so 2x + 1 = 0 or x + 2 = 0 and therefore x = -1 (rather than -½ or x = -2)
* Some pupils may not divide a quadratic equation by a common factor to make the factorising process easier, such as 2x2 + 6x + 8 = 0
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| *Analysing statistics* | *2 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Analyse distributions of data sets
 | * Understand the limitations of sampling
* Use a sample to infer properties of a population
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the meaning of discrete and continuous data
* Interpret and construct frequency tables
* Analyse data using measures of central tendency
 | Categorical data, Discrete dataContinuous data, Grouped dataAxis, axesPopulationSampleCentral tendencyMean, median, modeSpread, dispersion, consistency**Notation**Correct use of inequality symbols when labeling groups in a frequency table | In Stage 8, pupils explore how to find the modal class of set of grouped data, the class containing the median of a set of data, the midpoint of a class, an estimate of the mean from a grouped frequency table and an estimate of the range from a grouped frequency tableNCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf) |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
|  | OCR: [Sampling](http://www.ocr.org.uk/Images/282254-foundation-topic-check-in-12.01-sampling.docx), [Analysing Data](http://www.ocr.org.uk/Images/289113-higher-topic-check-in-12.03-analysing-data.docx)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) |  |

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| *Mathematical movement II* | *5 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore the concept of a vector
* Solve problems involving vectors
 | * Understand the concept of a vector
* Know and use different notations for vectors, including diagrammatic representation
* Add and subtract vectors
* Multiply a vector by a scalar
* Solve simple geometrical problems involving vectors
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand column vector notation
 | VectorScalarConstantMagnitude**Notation*****a*** (print) and *a* (written) notation for vectors$\vec{AB}$ notation for vectorsColumn vector notation $\left(\begin{matrix}p\\q\end{matrix}\right)$, *p* = movement right and *q* = movement up | In Stage 7, pupils described a translation as a 2D vector. This unit is designed to explore vectors in more detail.Vector is a latin word for ‘carrier, transporter’ derived from veho (‘I carry, I transport, I bear’). Vectors have magnitude and direction.Scalar is from the latin ‘scala’ meaning ‘a flight of steps, stairs, staircase’.Scalars have magnitude but no direction.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches**Pupils either use underline notation, such as *a,*or $\vec{AB}$ notation when writing vectors***.*** |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of values for a and b to satisfy $\left(\begin{matrix}a\\2\end{matrix}\right)$ + 3$\left(\begin{matrix}b\\2\end{matrix}\right)= \left(\begin{matrix}10\\8\end{matrix}\right) .$ And another pair. And another pair.
* If $\vec{OA}$ = **a** and $\vec{OB}$ = **b** , convince me the vector $\vec{AB}$= **b** – **a**
* Always/Sometimes/Never: $\vec{AB}$=$-\vec{BA}$
 | KM: [Vectors](http://www.kangaroomaths.com/free_resources/autograph/010Getting%20Going/205addvectors3.20.doc)NRICH: [Vectors](http://nrich.maths.org/8753)CIMT: [Vectors](http://www.cimt.plymouth.ac.uk/projects/mepres/allgcse/bkc19.pdf)AQA: [Bridging Units: Vectors](http://allaboutmaths.aqa.org.uk/attachments/5627.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M12 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/10M12_BAM.pdf) | * Some pupils may try to write column vectors as fractions, i.e. $\left(\frac{1}{2}\right)$ instead of $\left(\begin{matrix}1\\2\end{matrix}\right)$
* If $\vec{OA}$ = **a** and $\vec{OB}$ = **b** , some pupils may calculate the vector $\vec{AB}$as **a** – **b**
* Some pupils may calculate 2$\left(\begin{matrix}a\\b\end{matrix}\right) $as $\left(\begin{matrix}2a\\b\end{matrix}\right)$
 |