**Mathematics overview: Stage 5**

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| *Unit* | *Hours* | *Mastery indicators* | *Essential knowledge* |
| [Numbers and the number system](#NNS) | 8 | * [Identify multiples and factors of a number](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M1_BAM.pdf)
* [Count forwards and backwards through zero](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M2_BAM.pdf)
* [Round to one decimal place](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M3_BAM.pdf)
* [Use columnar addition and subtraction with numbers of any size](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M4_BAM.pdf)
* [Multiply a three- or four-digit number by a two-digit number using long multiplication](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M5_BAM.pdf)
* [Divide numbers up to four-digits by a single-digit number using short division and interpret the remainder](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M6_BAM.pdf)
* [Add and subtract fractions with denominators that are multiples of the same number](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M7_BAM.pdf)
* [Write decimals as fractions](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M8_BAM.pdf)
* [Understand that per cent relates to number of parts per hundred](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M9_BAM.pdf)
* [Convert between adjacent metric units of measure for length, capacity and mass](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M10_BAM.pdf)
* [Measure and draw angles](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M11_BAM.pdf)
* [Calculate the area of rectangles](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M12_BAM.pdf)
* [Distinguish between regular and irregular polygons](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M13_BAM.pdf)
* [Stage 5 BAM Progress Tracker Sheet](http://kangaroomaths.com/free_resources/planning/stage5_tracker.pdf)
 | * Know the place value headings up to millions
* [Recall primes to 19](http://kangaroomaths.com/free_resources/display/number_classification.pdf)
* [Know the first 12 square numbers](http://kangaroomaths.com/free_resources/display/number_classification.pdf)
* Know the Roman numerals I, V, X, L, C, D, M
* Know the % symbol
* Know percentage and decimal equivalents for 1/2, 1/4, 1/5, 2/5, 4/5
* Know rough conversions between metric and Imperial units
* Know that angles are measured in degrees
* Know angles in one whole turn total 360°
* Know angles in half a turn total 180°
* [Know that area of a rectangle = length × width](http://kangaroomaths.com/free_resources/display/areas.pdf)
 |
| [Counting and comparing](#CC) | 8 |
| [Calculating: addition and subtraction](#CAS) | 8 |
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| *Numbers and the number system* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
* know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
* establish whether a number up to 100 is prime and recall prime numbers up to 19
* recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Identify multiples of numbers
* Explore factors of numbers
* Investigate prime numbers
* Work with square and cube numbers

**Bring on the Maths+: Moving on up!**Solving problems: #3 | * Know the meaning of ‘multiple’
* Identify multiples of a given number
* Know the meaning of ‘factor’
* Know how to find factors of a given number
* Know the meaning of ‘common factor’
* Know the meaning of ‘prime number’
* Recall the prime numbers less than 20
* Know how to test if a number up to 100 is prime
* Understand the use of notation for squared and cubed
* Work out the first 10 square numbers
* Work out the first 5 cube numbers
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recall multiplication facts to 12 × 12 and associated division facts
 | Multiple(Common) factorDivisibleFactor pairsPrime number, Composite numberSquare number, Cube numberPower**Notation**52 is read as ‘5 to the power of 2’ or ‘5 squared’ and means ‘2 lots of 5 multiplied together’53 is read as ‘5 to the power of 3’ or ‘5 cubed’ and means ‘3 lots of 5 multiplied together’ | ‘Squared’ and ‘cubed’ are special cases of powers. The language ‘to the power of’ can also be introduced to prepare pupils for the future when they will deal with higher powers.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.**Every classroom has a set of* [*number classification posters*](http://kangaroomaths.com/free_resources/display/number_classification.pdf) *on the wall* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Kenny says ’16 is a square number because 82 = 16’. Explain why Kenny is wrong.
* Convince me that 17 is a prime number
* Show me an example of a multiple of 4. And another. Now find a multiple of 4 that you think no one else in the room will choose.

NCETM: [Multiplication and Division Reasoning](https://www.ncetm.org.uk/public/files/18438909/3_Progression_Map_Multiplication_and_Division_Reasoningv2.pdf) | KM: [Dominoes](http://kangaroomaths.com/free_resources/teaching/number/dominoes.pptx). Use the scoring system.KM: Use [Eratosthenes' sieve](http://kangaroomaths.com/free_resources/teaching/number/eratosthenes_sieve.xlsx) to identify prime numbers up to 100KM: [Exploring primes activities](http://kangaroomaths.com/free_resources/teaching/number/exploring_primes.docx): Numbers of factorsKM: [Square numbers](http://kangaroomaths.com/free_resources/teaching/number/square_numbers.pptx)NRICH: [Factors and multiples KS2](http://nrich.maths.org/8960)NRICH: [Two primes make one square](http://nrich.maths.org/1150)NRICH: [Up and down staircases](http://nrich.maths.org/2283)**Learning review**KM: [5M1 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M1_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as ‘a number which is divisible by itself and 1’
* A common misconception is to believe that 62 = 6 × 2 = 12
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| *Counting and comparing* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
* read Roman numerals to 1000 (M) and recognise years written in Roman numerals
* interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Work with numbers up to one million
* Understand and use Roman numerals
* Understand and use negative numbers

**Bring on the Maths+: Moving on up!**Number and Place Value: #1 | * Understand place value in numbers with up to seven digits
* Order numbers up to and including those with seven digits
* Write numbers up to and including those with seven digits
* Read numbers up to and including those with seven digits
* Know the meaning of the Roman numerals D and M
* Interpret a year when written in Roman numerals
* Count backwards in whole number steps when negative numbers are included
* Count forwards in whole number steps when negative numbers are included
* Understand and use temperatures below 0°C
* Interpret negative numbers in other contexts
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand and use place value in four-digit numbers
* Know Roman numerals from I to C
* Read numbers written in Roman numerals up to 100
* Count forwards and backwards in whole number steps
 | Place valueDigitRoman numeralsNegative number**Notation**See notes about Roman numerals | Zero is neither positive nor negative.Ensure that pupils read information carefully and check whether the required order is smallest first or greatest first.Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g. 1 029 628)In general it is incorrect to repeat a Roman numeral symbol four times (i.e. XXXX). Also, the subtractive method should only be used (1) if subtracting powers of ten (i.e. I, X or C), and (2) if subtracting from the next two higher symbols (for example, I can be subtracted from V or X, but not L, C, D or M). Therefore 49 cannot be written as XXXXIX, or as IL, and must be written as XLIX. See NCETM: [Roman numerals](https://www.ncetm.org.uk/resources/11689) NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers use the language ‘negative number’, and not ‘minus number’, to avoid future confusion with calculations**Every classroom has a* [*negative number washing line*](http://kangaroomaths.com/free_resources/display/number_line.docx) *on the wall**Every classroom displays a number line up to 1 000 000**Every classroom has a place value chart on the wall* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Look at this number (1 029 628). Show me another number (with 4, 5, 6, 7 digits) that includes a 9 with the same value. And another. And another …
* Jenny reads the number 1 029 008 as ‘one million, twenty nine thousand and eight’. Kenny reads the same number as ‘one million, two hundred and nine thousand and eight’. Who is correct? How do you know?
* Convince me that 2014 is MMXIV in Roman numerals
* Convince me that -17°C is colder than -14°C

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | KM: [Roman numeral converter](http://kangaroomaths.com/free_resources/teaching/number/arabic_roman_converter.xlsx). Note that we use Arabic numerals today! Choose a number and convert it instantly. Can pupils work out the system for numbers above 100?KM: [Roman numeral times table jigsaw](http://kangaroomaths.com/free_resources/teaching/number/tables_jigsaw.xlsx): Use the larger version to start looking at numbers above 100.NRICH: [Sea level](http://nrich.maths.org/5929)NRICH: [Tug Harder!](http://nrich.maths.org/public/viewer.php?obj_id=5898)**Learning review**KM: [5M2 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M2_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils can confuse the language of large (and small) numbers since the prefix ‘milli- means ‘one thousandth’ (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.
* The use of IIII on a clock face suggests that a Roman numeral can be repeated four times, but this is a special case. In general, three is the maximum number of repeats and the subtractive method should be used instead (i.e. IV)
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| *Calculating: addition and subtraction* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * add and subtract numbers mentally with increasingly large numbers
* add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
* solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Develop mental addition and subtraction skills
* Extend written methods of addition and subtraction
* Solve problems involving addition and subtraction
 | * Add four-digit numbers and ones, tens, hundreds or thousands mentally
* Subtract four-digit numbers and ones, tens, hundreds or thousands mentally
* Add a three-digit number to a two-digit number mentally (when no bridging of hundreds is required)
* Use column addition for numbers with more than four digits
* Use column subtraction for numbers with more than four digits
* Identify when addition or subtraction is needed as part of solving multi-step problems
* Explain why addition or subtraction is needed at any point when solving multi-step problems
* Solve multi-step problems involving addition and/or subtraction
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Add and subtract numbers mentally, including a three-digit number and ones, tens or hundreds
* Use column addition and subtraction for numbers up to four digits
* Estimate the answer to a calculation

**Bring on the Maths+: Moving on up!**Calculating: #1 | AdditionSubtractionSum, TotalDifference, Minus, LessColumn additionColumn subtractionExchangeOperationEstimate | Ensure that pupils can deal with column subtractions that include a 0 within the first number; e.g. 48027 – 8437.Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.KM: [Progression: Addition and Subtraction](http://kangaroomaths.com/free_resources/hod/bouncebuzz_addition_subtraction_v4.pdf) and [Calculation overview](http://kangaroomaths.com/free_resources/hod/bouncebuzz_calculation_overview_v4.pdf)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44567), [Subtraction](https://www.ncetm.org.uk/resources/40532)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***To avoid confusion with language, all teachers use ‘sum’ to refer only to the result of an addition. Teachers say ‘complete these calculations’ instead of ‘complete these sums’**All pupils use books / paper with 7mm squares and ensure that each digit is written in one square**When carrying, those numbers being carried are placed beneath the answer line**During column subtraction the language of ‘exchanging’ is used instead of ‘borrowing’. When exchanging, those numbers being altered or moved are written above the calculation*  |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Provide examples of column addition and subtraction with hidden digits. Challenge pupils to find these digits and explain their reasoning.
* Show me an example of a column addition (that includes carrying) with the answer 54192
* Convince me that 56095 – 23622 = 32473

NCETM: [Addition and Subtraction Reasoning](https://www.ncetm.org.uk/public/files/18416326/2_Progression_Map_Addition_and_Subtraction_Reasoning.pdf) | KM: [Palindromic numbers](http://kangaroomaths.com/free_resources/teaching/number/palindromic_numbers.docx)KM: [The Heinz matrix](http://kangaroomaths.com/free_resources/teaching/number/heinz2.docx). Tasks 1 and 2.KM: [Pairs in squares](http://kangaroomaths.com/free_resources/teaching/number/pairs_in_squares.docx)KM: [Interactive target boards](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_Interactive_Target_Boards.xlsm)KM: [Maths to Infinity: Addition and subtraction foundations](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_TheFoundations_AddSub.xlsm)NRICH: [Journeys in Numberland](http://nrich.maths.org/7285)NRICH: [Twenty Divided Into Six](http://nrich.maths.org/public/viewer.php?obj_id=1047)NRICH: [Two and Two](http://nrich.maths.org/public/viewer.php?obj_id=781)KM: Following on from ‘Two and Two’ above, why is FIVE + TWO = SEVEN impossible? How about THREE + NINE = TWELVE and FORTY + FORTY = EIGHTY? Consider column methods.**Learning review**KM: [5M4 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M4_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
* Some pupils incorrectly assume and use commutativity within column subtraction; for example:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 7 | 4 | 1 | 2 | 6 |
| – | 2 | 3 | 7 | 3 | 4 |
|  | 5 | 1 | 6 | 1 | 2 |

* Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)
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| *Calculating: multiplication and division* | *12 hours* |
| **Key concepts** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * multiply and divide numbers mentally drawing upon known facts
* multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
* multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
* divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
* solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
* solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates
* solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Develop mental arithmetic skills
* Explore multiplication and division of decimals
* Develop written methods of multiplication
* Develop written methods of division
* Solve problems involving multiplication and division

**Bring on the Maths+: Moving on up!**Calculating: #3Solving problems: #1 | * Use knowledge of multiplication tables when multiplying and dividing mentally
* Multiply (or divide) a whole number or decimal by 10, 100 or 1000
* Know how to set up a long multiplication problem
* Use long multiplication to a multiply a two- (or three-, or four-) digit number by a two-digit number
* Understand the method of short division
* Use short division to divide a two- (or three-, or four-) digit number by a one-digit number
* Interpret a remainder when carrying out division
* Identify the correct operation(s) required in order to solve a problem
* Identify when knowledge of factors (multiples, squares, cubes) can be used to help solve a problem
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recall multiplication facts for multiplication tables up to 12 × 12
* Recall division facts for multiplication tables up to 12 × 12
* Find factor pairs of a given number
* Understand the commutativity of multiplication
* Multiply and divide a two-digit number by 10, 100
* Multiply a three-digit number by a one-digit number using short multiplication
 | Multiply, Multiplication, Times, ProductCommutativeDivide, DivisionDivisibleDivisor, Dividend, Quotient, RemainderFactorShort multiplicationLong multiplicationShort divisionOperationEstimate**Notation**Remainders are often abbreviated to ‘r’ or ‘rem’ | The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.KM: [Progression: Multiplication and Division](http://kangaroomaths.com/free_resources/hod/bouncebuzz_multiplication_division_v4.pdf) and [Calculation overview](http://kangaroomaths.com/free_resources/hod/bouncebuzz_calculation_overview_v4.pdf)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44568), [Multiplication](https://www.ncetm.org.uk/resources/40530), [Division](https://www.ncetm.org.uk/resources/43589), [Multiplicative reasoning](https://www.ncetm.org.uk/resources/43669)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All classrooms display a* [*times table poster with a twist*](http://kangaroomaths.com/free_resources/display/chinese_tables.docx)*To avoid confusion with language, all teachers use ‘sum’ to refer only to the result of an addition; e.g. ‘complete these calculations’ (not ‘sums’).**The use of long multiplication is promoted as the ‘most efficient method’.* *Short division is promoted as the ‘most efficient method’.**When dealing with remainders in division problems, use the notation ‘r’* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Find missing digits in otherwise completed long multiplication / short division calculations
* Convince me that 247 × 12 = 2964
* What is the same and what is different: 1344 × 6 and 504 × 16?
* What is wrong with this short division? How can you correct it?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 0 | 10 | 7 | r 5 |
| 8 | 3 | 86 | 61 |  |

NCETM: [Multiplication and Division Reasoning](https://www.ncetm.org.uk/public/files/18438909/3_Progression_Map_Multiplication_and_Division_Reasoningv2.pdf) | KM: [Happy and sad](http://kangaroomaths.com/free_resources/teaching/number/happy_sad.docx)KM: [Short multiplication](http://www.kangaroomaths.com/free_resources/teaching/number/short_multiplication.xlsx)KM: [Long multiplication template](http://kangaroomaths.com/free_resources/teaching/number/long_multiplication_template.docx)KM: [Maximise, minimise](http://kangaroomaths.com/free_resources/teaching/number/maxmin.docx). Game 2.KM: [Tens and hundreds](http://kangaroomaths.com/free_resources/teaching/number/tens_hundreds.docx). Use [Powers of ten](http://kangaroomaths.com/free_resources/teaching/number/powers_of_ten.xlsx) to demonstrate connections.KM: [Maths to Infinity: Multiplying and dividing](http://kangaroomaths.com/free_resources/infinity/multiply_divide.xlsm)KM: [Interactive target boards](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_Interactive_Target_Boards.xlsm)KM: [Maths to Infinity: Multiplication and division foundations](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_The%20Foundations_MultDiv.xlsm)NRICH: [Curious Number](http://nrich.maths.org/7218)NRICH: [Make 100](http://nrich.maths.org/public/viewer.php?obj_id=1013)NRICH [Dicey Operations](http://nrich.maths.org/6606). Games 4 and 5.**Learning review**KM: [5M5 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M5_BAM.pdf), KM: [5M6 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M6_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may write statements such as 2 ÷ 8 = 4
* When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 0 | 10 | 7 | r 5 |
| 8 | 3 | 86 | 61 |  |

3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been ‘moved across’ and therefore everything that follows has been correctly carried out based on an early misunderstanding. |

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| *Investigating properties of shapes* | *4 hours* |
| **Key concepts** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * use the properties of rectangles to deduce related facts and find missing lengths and angles
* distinguish between regular and irregular polygons based on reasoning about equal sides and angles
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Explore the properties of rectangles
* Investigate polygons
 | * Know the definition and properties of a rectangle
* Use the properties of rectangles to find missing lengths and angles
* Use the properties of rectangles to find points on a coordinate grid
* Know the definition of a polygon
* Know the difference between a regular and an irregular polygon
* Identify whether or not a polygon is regular
* Use the properties of polygons to find missing lengths and angles
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Identify right angles
* Use coordinates in the first quadrant

**Bring on the Maths+: Moving on up!**Position and direction: #2 | RectangleSquareQuadrilateral(Regular / irregular) polygon, pentagon, hexagon, octagon(Right) angleParallelPerpendicularCoordinates**Notation**Dash notation to represent equal lengths in shapes and geometric diagramsRight angle notation(Cartesian) coordinates | Note that a square is a rectangle but a rectangle is not necessarily a square.A square is a regular quadrilateral.Pupils may also know names of other polygons such as heptagon (7 sides), nonagon (9 sides), decagon (10 sides) and dodecagon (12 sides).NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom has a set of* [*triangle posters*](http://kangaroomaths.com/free_resources/display/triangles.pdf) *and* [*quadrilateral posters*](http://kangaroomaths.com/free_resources/display/quadrilaterals.pdf) *on the wall* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that a square is a rectangle
* Show me an example of a hexagon. And another, and another, …
* What is the same and what is different:

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Shape work](http://kangaroomaths.com/free_resources/teaching/geometry/shape_work.docx): Rectangle, Packing squaresNRICH: [Egyptian Rope](http://nrich.maths.org/public/viewer.php?obj_id=982)NRICH: Use the [virtual geoboard](http://nrich.maths.org/2883) to explore how regular polygons can be made using equally spaced points around a circle, and ways of constructing rectangles on any of the three type of boardKM: [6 point circles](http://kangaroomaths.com/free_resources/teaching/geometry/6_point_circles.pdf), [8 point circles](http://kangaroomaths.com/free_resources/teaching/geometry/8_point_circles.pdf) and [12 point circles](http://kangaroomaths.com/free_resources/teaching/geometry/12_point_circles.pdf) can be used to support the above idea**Learning review**KM: [5M13 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M13_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that a ‘regular’ polygon is a ‘normal’ polygon
* Some pupils may think that all polygons have to be regular
* Some pupils may use coordinates the wrong way round; for example, interpreting the point (3,2) as 3 up and 2 across (to the right)
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| *Visualising and constructing* | *4 hours* |
| **Key concepts** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * identify 3-D shapes, including cubes and other cuboids, from 2-D representations
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Investigate 3D shapes
 | * Identify 3D shapes from photographs
* Identify 3D shapes from sketches
* Identify 3D shapes from nets
* Identify 3D shapes from diagrams on isometric paper
* Construct diagrams of 3D shapes on isometric paper
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the names of common 3D shapes
 | CubeCuboidCylinderPyramidPrismConeSphere2D3DNetSketchIsometric paper | A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid.A cube is a special case of a cuboid, and a cuboid is a special case of a prism.Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom has a set of* [*3D shape posters*](http://kangaroomaths.com/free_resources/display/solids.pdf) *on the wall**Models of 3D shapes to be used by all students during this unit of work* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * (Showing photograph / sketch / isometric drawing / net), convince me that this shape is a cuboid / cube / prism / …
* Show me a way to draw a cube. And another. And another …
* Show me a way to draw a 2cm by 3cm by 4cm cuboid on isometric paper. And another. And another …
* What is wrong with this sketch of a cuboid? How can it be changed?

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Shape work](http://kangaroomaths.com/free_resources/teaching/geometry/shape_work.docx): Dice, Opposite numbers, Cutting cubes, Painted cubeNRICH: [The Third Dimension](http://nrich.maths.org/1148)NRICH: [A Puzzling Cube](http://nrich.maths.org/1140)NRICH: [Rolling That Cube](http://nrich.maths.org/7299)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Pupils must have isometric paper in portrait orientation for it to work correctly.
* When drawing a cube on isometric paper, some students may think that they need to join dots to make a square first, and will draw horizontal and vertical lines to attempt to achieve this
* Correct use of isometric paper must not indicate ‘hidden’ lines
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| *Exploring time* | *4 hours* |
| **Key concepts** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * solve problems involving converting between units of time
* complete, read and interpret information in tables, including timetables
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Solve problems involving time
* Interpret information in tables
* Interpret information in timetables

**Bring on the Maths+: Moving on up!**Measures: #1, #2 | * Identify when it is necessary to convert between units of time to solve a problem
* Choose a correct conversion to use
* Convert a given time into a different unit of time
* Solve a problem involving converting between different units of time
* Identify the types of information arranged in a table
* Read information given in a table
* Interpret the meaning of information given in a table
* Interpret the meaning of information given in a timetable
* Complete a table from given information
* Solve problems that involve interpreting timetables
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Read, write and convert time between analogue and digital 12- and 24-hour clocks
* Know how to convert from hours to minutes; minutes to seconds; years to months; weeks to days
 | MillenniumCenturyDecadeYearMonthWeekDayHourMinuteSecondTimetable**Notation**12- and 24-hour clock notation24-hour clock notation can be with or without a colon separating hours and minutesAnalogue clocks with Arabic or Roman numerals | NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils solve problems involving the use of local bus and train timetables* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * (Using a timetable) I want to arrive in Chichester by 10:15. Show me a train that I could catch from Portsmouth. And another. What is the latest train I could catch? What time does this train leave Portsmouth?
* Convince me that that are 135 minutes between 1115 and 1:30 p.m.
* Jenny and Kenny are solving a problem that involves planning a journey. They are leaving Chester at 08:12. The journey takes 1 hour and 50 minutes. Jenny thinks that they will arrive at 09:62. Kenny thinks that they will arrive at 10:02. Who do you agree with? Explain your answer.

NCETM: [Measurement Reasoning](https://www.ncetm.org.uk/public/files/18436766/7_Progression_Map_Measurement_Reasoning.pdf) | KM: [Timetable progression](http://kangaroomaths.com/free_resources/teaching/geometry/timetable_progression.docx)NRICH: [Watch the clock](http://nrich.maths.org/public/viewer.php?obj_id=980)NRICH: [Two clocks](http://nrich.maths.org/public/viewer.php?obj_id=4806)NRICH: [Train timetable](http://nrich.maths.org/958)NRICH: [Slow coach](http://nrich.maths.org/1162)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
* Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
* Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o’ clock
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| *Exploring fractions, decimals and percentages* | *12 hours* |
| **Key concepts** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * compare and order fractions whose denominators are all multiples of the same number
* identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
* recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
* read and write decimal numbers as fractions [for example, 0.71 = 71/100]
* read, write, order and compare numbers with up to three decimal places
* recognise the per cent symbol (%) and understand that per cent relates to ‘number of parts per hundred’, and write percentages as a fraction with denominator 100, and as a decimal
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Explore the equivalence between fractions
* Explore the equivalence between fractions and decimals
* Understand the meaning of percentages
 | * Use diagrams to compare the size of fractions
* Identify when two fractions can be compared easily without a diagram
* Compare two fractions without using a diagram
* Order a set of fractions when the denominators are equal
* Know that fractions can be equivalent
* Know how use a diagram to find a fraction that is equivalent to another fraction
* Know that thousandths is the name of the column beyond hundredths
* Understand that thousandths can be written as fractions or as decimals
* Write a number (less than1) with one decimal place as a fraction
* Write a number (less than 1) with two decimal places as a fraction
* Read a number with three decimal places
* Compare a set of numbers written to three decimal places
* Compare a set of numbers with a mixed number of decimal places
* Understand that per cent relates to number of parts per hundred
* Understand that a percentage can be written as a fraction with a denominator of 100
* Write any percentage as a decimal
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of equivalent fractions
* Understand that tenths and hundredths can be written as fractions or as decimals
* Know that 1/4 = 0.25, 1/2 = 0.5 and 3/4 = 0.75
 | FractionNumeratorDenominatorImproper fraction, Proper fraction, Vulgar fraction, Top-heavy fractionTenth, hundredth, thousandthPer cent, PercentageDecimalEquivalent**Notation**Diagonal fraction bar / horizontal fraction bar | NRICH: [Teaching fractions with understanding](http://nrich.maths.org/2550)NCETM: [Teaching fractions](https://www.ncetm.org.uk/resources/44490)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are made aware that ‘per cent’ is derived from Latin and means ‘out of one hundred’**Teachers use the horizontal fraction bar notation at all times* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a fraction that is equivalent to 7/10. And another …
* Convince me that 6/8 is greater than 7/16
* Jenny says that 0.127 is ‘one hundred and twenty seven thousandths’. Kenny says that 0.127 is ‘one tenth, two hundredths and seven thousandths’. Who do you agree with? Explain your reasoning.

NCETM: [Fractions Reasoning](https://www.ncetm.org.uk/public/files/18416412/4_Progression_Map_Fractions_Reasoning_.pdf) | KM: [Decimal ordering cards 1](http://kangaroomaths.com/free_resources/teaching/number/decimal_ordering_cards_1.docx)KM: [Fraction action](http://kangaroomaths.com/free_resources/teaching/number/fraction_action.docx)KM: [Carpets](http://kangaroomaths.com/free_resources/teaching/number/fdp_carpets.docx)NRICH: [Spiralling decimals](http://nrich.maths.org/10326)NCETM: [Activity D - Metre sticks and metre strips](https://www.ncetm.org.uk/resources/42655)NCETM: [Activity F - Using blank hundred squares](https://www.ncetm.org.uk/resources/42655)**Learning review**KM: [5M8 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M8_BAM.pdf), KM: [5M9 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M9_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may read 0.234 as ‘nought point two hundred and thirty four’. This leads to the common misconception that, for example, 0.400 is a number larger than 0.76
* Pupils may not make the connection that a percentage is a different way of describing a proportion
* Some pupils may think that equivalent fractions are found using an additive relationship rather than a multiplicative one: for example, that the fraction 4/5 is equivalent to 6/8
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| *Pattern sniffing* | *4 hours* |
| **Key concepts** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Develop ways of counting
 | * Count forwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000)
* Count backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000)
* Count forwards through zero
* Count backwards through zero
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand place value in numbers with up to seven digits
* Read and write numbers up to and including those with seven digits
* Count backwards in whole number steps when negative numbers are included
* Count forwards in whole number steps when negative numbers are included
 | ForwardsBackwardsAscendingDescendingPatternSequence | Pupils have counted forwards and backwards in previous years and units, but this is the first time that ‘Pattern Sniffing’ appears as a unit in its own right. NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers and pupils refer to numbers less than zero as ‘negative’ numbers and not ‘minus’ numbers* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a number that is easy (difficult) to count forward in tens (hundreds, thousands). And another. And another …
* Kenny is counting forwards ... ‘*4060, 4070, 4080, 4090, 5000*.’ Do you agree with Kenny? Explain your answer.
* Convince me that one less than -2 is -3 and not -1

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | NRICH: [Tug Harder!](http://nrich.maths.org/public/viewer.php?obj_id=5898)NCETM: [Activity set B](https://www.ncetm.org.uk/resources/42499)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think the negative number line is:

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 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10* Some pupils may bridge straight to the next thousand rather than the next hundred, such as ‘*4060, 4070, 4080, 4090, 5000*.’
* Some pupils may think that 1million is one more than 9999.
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| *Measuring space* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
* understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints
* use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Convert between measures
* Know and work with common Imperial units
* Solve problems involving measurement
* Solve problems involving money
 | * Convert between centimetres and metres
* Convert between kilograms and grams
* Use decimal notation when converting between metric units of length, mass and volume / capacity
* Know that one inch is roughly equivalent to 2.5 cm
* Know that one foot is roughly equivalent to 30 cm
* Know that one kilogram is roughly equivalent to 2.2 lb
* Know that one pint is roughly equivalent to 550 ml
* Use rough equivalents between metric and Imperial units when solving problems
* Choose the correct operations when solving problems involving measures
* Solving scaling problems involving measure
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Convert between kilometres and metres, centimetres and millimetres
* Convert between litres and millilitres
* Convert between hours and minutes, minutes and seconds
* Use decimal notation to two decimal places when converting between measures
 | Length, distanceMass, weightVolumeCapacityMetre, centimetre, millimetreKilogram, gramLitre, millilitreHour, minute, secondInch, foot, yardPound, ouncePint, gallon**Notation**Abbreviations of units in the metric system: m, cm, mm, kg, g, l, mlAbbreviations of units in the Imperial system: lb, oz | Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement.The prefix ‘centi-‘ means one hundredth, and the prefix ‘milli-‘ means one thousandth. These words are of Latin origin.The prefix ‘kilo-‘ means one thousand. This is Greek in origin.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Kenny thinks 1.5m = 105cm. Do you agree with Kenny? Explain your answer
* Show me an imperial (metric) unit of measure. And another. And another.
* Convince me that 3.07kg = 3070g.
* Which of the following is the best value for money?

1 litre for £2 or 2 pints for £25kg for 40p or 4lbs for 40p10cm for £2 or 5 inches for £2NCETM: [Measurement Reasoning](https://www.ncetm.org.uk/public/files/18436766/7_Progression_Map_Measurement_Reasoning.pdf) | NRICH: [Olympic Starters](http://nrich.maths.org/8170)NCETM: [Activity D - Converting between metric units](https://www.ncetm.org.uk/resources/42796)NCETM: [Activity E- Converting between metric and imperial](https://www.ncetm.org.uk/resources/42796)**Learning review**KM: [5M10 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M10_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30.
* Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an ‘extra unit’ within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.
* Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
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| *Investigating angles* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
* draw given angles, and measure them in degrees (°)
* identify: angles at a point and one whole turn (total 360°); angles at a point on a straight line and 1/2 a turn (total 180°); other multiples of 90°
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Develop knowledge of angles
* Measure angles
* Draw angles
 | * Know that angles are measured in degrees
* Know that angles in a full turn total 360°
* Know that angles on a straight line total 180°
* Know that a reflex angle is greater than 180°
* Identify angles at a point
* Identify angles at a point on a straight line
* Estimate the size of angles
* Use a protractor to measure angles less than 180°
* Use a protractor to measure angles greater than 180°
* Use a protractor to draw angles less than 180°
* Use a protractor to draw angles greater than 180°
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand that an acute angle is less than a right angle
* Understand that an obtuse angle is greater than a right angle and less than two right angles
* Identify acute angles
* Identify obtuse angles
* Identify acute, obtuse and right angles in shapes
* Compare angles up to two right angles in size
* Order angles up to two right angles in size
 | TurnAngleDegreesRight angleAcute angleObtuse angleReflex angleProtractor**Notation**Right angle notationArc notation for all other anglesThe degree symbol (°) | The use of degrees as a unit for measuring angles is first introduced in this unit.The exact reason for there being 360 degrees in a full turn is unknown. There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors.The SI unit for measuring angles in the radian (2π radians in a full turn). Napoleon experimented with the decimal degree, or grad (400 grads in a full turn)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are taught to use a 180° and a 360° protractor.**Teachers reference the Babylonian number system for explaining why there are 360° in one whole turn.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an acute (obtuse, reflex) angle. And another. And another.
* Jenny uses a protractor to measure this angle:

 She writes down 140°. Do you agree with  Jenny?* Convince me how to measure a reflex angle using a 180° protractor.
* Kenny thinks that 90° is an acute angle. Jenny thinks that 90° is an obtuse angle. Who is correct? Explain your answer.

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Angle Vocab](http://kangaroomaths.com/free_resources/ks3/resources/MAP/angle_vocab.doc)NRICH: [Estimating Angles](http://nrich.maths.org/1235)NCETM: [Activity A: Logo Challenge 1 – Star Square](https://www.ncetm.org.uk/resources/42849)NCETM: [Activity C: Equal angles](https://www.ncetm.org.uk/resources/42849)NCETM: [Activity D: Sorting triangles](https://www.ncetm.org.uk/resources/42849)**Learning review**KM: [5M11 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M11_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils use the wrong scale on a protractor. For example, they measure an obtuse angle as 60° rather than 120°.
* Some pupils may think that 90° is either an acute or obtuse angle.
* Some pupils may think it is not possible to measure a reflex angle.
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| *Calculating fractions, decimals and percentages* | *12 hours* |
| **Key concepts** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 2/5 + 4/5 = 6/5 = 1 1/5]
* add and subtract fractions with the same denominator and denominators that are multiples of the same number
* multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
* solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25
* solve problems involving number up to three decimal places
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Explore mixed numbers
* Calculate with fractions
* Explore fractions, decimals and percentages

**Bring on the Maths+: Moving on up!**Fractions, decimals & percentages: #5 | * Understand the concept of a mixed number
* Convert a mixed number into an improper fraction (and vice versa)
* Add (subtract) fractions when one denominator is a multiple of the other
* Multiply a proper fraction (mixed number) by a whole number
* Use diagrams to explain multiplication of a proper fraction or mixed number by a whole number
* Write the answer to a calculation as a mixed number when appropriate
* Know percentage equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and fractions with a denominator of 10 and 100
* Establish percentage equivalents of fractions with a denominator of 20, 25, 40 and 50
* Know decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and fractions with a denominator of 10 and 100
* Establish decimal equivalents of fractions with a denominator of 20, 25, 40 and 50
* Recognise that thousandths arise from dividing a number (or object) into one thousand equal parts
* Recognise that thousandths arise from dividing hundredths by ten
* Solve problems involving number up to three decimal places
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of an improper fraction
* Add and subtract fractions with the same denominator within and beyond one whole
* Recognise and use tenths and hundredths
* Understand that per cent relates to number of parts per hundred
* Understand that a percentage can be written as a fraction with a denominator of 100
* Write any percentage as a decimal
 | Place valueTenth, hundredth, thousandthDecimalProper fraction, Improper fraction, top-heavy fraction, vulgar fractionNumerator, denominatorPercent, percentage**Notation**Decimal pointt, h, th notation for tenths, hundredths, thousandthsHorizontal / diagonal bar for fractions | Describe 1/3 as ‘there are three equal parts and I take one’, and 3/4 as ‘there are four equal parts and I take three’.Be alert to pupils reinforcing misconceptions through language such as ‘the bigger half’.To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded.Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.NCETM: [Teaching fractions](https://www.ncetm.org.uk/resources/44490), [Fractions videos](https://www.ncetm.org.uk/resources/43609), [The Bar Model](https://www.ncetm.org.uk/resources/44565)**Common approaches***Teachers use the horizontal fraction bar notation at all times.**Pupils are encouraged to convert mixed numbers to improper fractions when subtracting.*  |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an improper fraction (mixed number). And another.
* Kenny thinks that $\frac{1}{4}+ \frac{2}{8}= \frac{3}{12}$ . Explain why Kenny is incorrect.
* Jenny thinks that you can only add or subtract fractions if they have the same common denominator. Do you agree with Jenny? Explain.
* Show me a fraction, decimal and percentage ‘equivalent family’ (e.g. $\frac{1}{2}$ = 50% = 0.5). And another. And another …
* Kenny thinks that $\frac{1}{3}×5=\frac{5}{15} $. Do you agree with Kenny? Explain.
* Convince me that $2\frac{2}{3}× 3=8$ in at least 2 different ways.

NCETM: [Fractions Reasoning](https://www.ncetm.org.uk/public/files/18416412/4_Progression_Map_Fractions_Reasoning_.pdf) | KM: [The Heinz Matrix 2](http://kangaroomaths.com/free_resources/teaching/number/heinz2.docx)NRICH: [Balance of Halves](https://nrich.maths.org/5677)NRICH: [Route Product](http://nrich.maths.org/public/viewer.php?obj_id=5632) NRICH: [Forgot the Numbers](http://nrich.maths.org/public/viewer.php?obj_id=1015) NCETM: [Activity A - Fractions ITP](https://www.ncetm.org.uk/resources/42655)**Learning review**KM: [5M7 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M7_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that you simply add the numerators and add the denominators when adding fractions.
* Some pupils may think that you simply subtract the numerators and subtract the denominators when subtracting fractions.
* Some pupils may think that you simply multiply both the numerator denominator when multiplying a fraction by a whole number.
* Some pupils may think that you simply multiply the whole number and then the fraction when multiplying a mixed number by a whole number, e.g. $3\frac{3}{4}× 2=6\frac{6}{4}$
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| *Calculating space* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
* calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm²) and square metres (m²) and estimate the area of irregular shapes
* estimate volume [for example, using 1 cm³ blocks to build cuboids (including cubes)] and capacity [for example, using water]
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Exploring the perimeter of composite shapes
* Calculate areas of rectangles
* Investigate volume and capacity

**Bring on the Maths+: Moving on up!**Measures: #4, #5 | * Calculate the perimeter of composite rectilinear shapes when some dimensions are unknown
* Know that the area of a rectangle is given by the formula area = length × width
* Know that area can be measured using square centimetres or square metres
* Know the abbreviations cm2 and m2
* Calculate the area of a rectangle
* Calculate the area of a square
* Understand how to estimate the area of irregular shapes
* Estimate the area of irregular shapes bounded by straight lines
* Estimate the area of irregular shapes that include curved lines
* Understand the concept of volume
* Understand the concept of capacity
* Estimate volume by using 1 cm3 blocks to build cuboids
* Estimate capacity
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of area
* Understand the concept of perimeter
* Calculate the perimeter of 2D shapes when dimensions are known
* Find the area of rectilinear shapes by counting squares
 | PerimeterAreaVolumeCapacityDimensionsSquare, rectangleComposite rectilinearPolygonCube, cuboidMillimetre, Centimetre, Metre, KilometreSquare centimetre, square metreCubic centimetre, centimetre cubeSquare unit**Notation**Abbreviations of units in the metric system: km, m, cm, mm, cm2, m2, cm3 | In terms of perimeter, this unit focuses solely on the composite rectilinear shapes; i.e. those that are bounded by straight lines that meet at right angles, see NCETM: [Y5 Measurement exemplification](https://www.ncetm.org.uk/resources/42798)This unit covers three concepts that pupils often confuse. It would be unwise to have a single objective for a lesson (or lessons) covering both area and perimeter for example.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils are taught to use the ‘matching method’ (see reasoning section) when estimating area of irregular shapes.**When estimating areas of irregular shapes pupils are taught to* *Pupils cut out and ‘feel’ the size of one square centimetre. They make a square metre using metre sticks and have the opportunity to visualise the fact that 10000 cm2 are equivalent to 1 m2.**Pupils make and ‘feel’ the size of one centimetre cube.**Pupils use 12 one-metre sticks to create and ‘feel’ the size of a cubic metre. They have the opportunity to visualise the fact that 1 000 000 cm3 are equivalent o 1 m3.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Jenny estimates the area of an irregular shape by counting all whole squares, and then matching up part squares to make whole squares. Benny estimates the area of the same shape by counting all whole squares and all squares that are mostly within the shape. He ignores squares mostly outside the shape. Whose method is best? Explain.
* Convince me that area of a rectangle = length × width
* Show me a shape with an area of 23 cm2. And another, and another …

NCETM: [Geometry -Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Stick on the Maths SSM7: Area and perimeter](http://www.kangaroomaths.com/free_resources/teaching/sotm/level5/5ssm7_ewb.doc)NRICH: [Area and Perimeter](http://nrich.maths.org/7280)NRICH: [Through the Window](https://nrich.maths.org/10344)NRICH: [Numerically Equal](http://nrich.maths.org/public/viewer.php?obj_id=1045)NRICH: [Cubes](http://nrich.maths.org/42)**Learning review**KM: [5M12 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M12_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may confuse the concepts of area and perimeter
* Some pupils may think that you multiply the numbers to find the perimeter of a shape.
* Some pupils may think that you cannot find the perimeter of a shape unless all the dimensions are given.
* Some pupils may just add the given dimensions, rather than consider any unlabelled dimensions
* Some pupils may think that you multiply all the numbers to find the area of a rectangle
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| *Checking, approximating and estimating* | *4 hours* |
| **Key concepts** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
* round decimals with two decimal places to the nearest whole number and to one decimal place
* use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Explore ways of approximating numbers
* Explore ways of checking answers
 | * Approximate any number by rounding to the nearest 10 000
* Approximate any number by rounding to the nearest 100 000
* Approximate any number with two decimal place by rounding to the nearest whole number
* Approximate any number with two decimal place by rounding to one decimal place
* Understand checking as the process of working backwards from the answer to ensure that it makes sense
* Understand estimating as the process of finding a rough value of an answer or calculation
* Estimate addition (subtraction) calculations with up to four digits
* Estimate multiplication calculations that involve multiplying up to four-digit numbers by a single digit
* Estimate multiplication calculations that involve multiplying up to four-digit numbers by a two-digit number
* Estimate division calculations that involve dividing up to a four-digit number by a single digit number
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Approximate any number by rounding to the nearest 10, 100 or 1000
* Approximate any number with one decimal place by rounding to the nearest whole number

**Bring on the Maths+: Moving on up!**Number and Place Value: #2 | Approximate (noun and verb)RoundDecimal placeCheckSolutionAnswerEstimate (noun and verb)AccurateAccuracy **Notation** The approximately equal symbol (≈) | This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.Also see big pictures: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) and [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are taught to visualise rounding through the use a number line* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that 150 000 rounds to 200 000 to the nearest 100 000
* What is the same and what is different: 1595, 1649, 1534 and 1634
* Benny thinks that 3.16 rounds to 3.1 to one decimal place. Do you agree? Explain your answer.

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | KM: [Stick on the Maths NNS2: Approximating](http://www.kangaroomaths.com/free_resources/teaching/sotm/level3/3nns2_ewb.doc)KM: [Maths to Infinity Rounding](http://www.kangaroomaths.com/free_resources/infinity/rounding.xlsm)NCETM: [Activity D](https://www.ncetm.org.uk/resources/42499)**Learning review**KM: [5M3 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/5M3_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may truncate instead of round
* Some pupils may misunderstand the rounding process as one that works from the end of the number; for example 3472 to the nearest 1000 is worked out as 3472 🡪 3470 🡪 3500 🡪 4000.
* Some pupils may round down at the half way point, rather than round up.
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| *Mathematical movement* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Use transformations to move shapes

**Bring on the Maths+: Moving on up!**Position and direction: #1 | * Identify a translation
* Carry out a translation described using mathematical language
* Identify a reflection
* Carry out a reflection using a mirror line parallel to the axes
* Carry out a reflection using a mirror line parallel to the axes and touching the object
* Carry out a reflection using a mirror line parallel to the axes and crossing the object
* Describe a reflection using mirror lines parallel to the axes
* Know the meaning of ‘congruent’, ‘congruence’, ‘object’, ‘image’
* Understand that a translation produces a congruent image
* Understand that a reflection produces a congruent image
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use coordinates in the first quadrant
* Describe a translation using mathematical language
 | 2-DGridAxis, axes, x-axis, y-axisOrigin(First) quadrant(Cartesian) coordinatesPointTranslationReflectionTransformationObject, ImageCongruent, congruence**Notation**Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y) | Note that pupils are not yet expected to use an algebraic description of a mirror line (such as x = 3).The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.Other coordinate systems include grid references, polar coordinates and spherical coordinates.There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is ‘transformations’.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers do not use the phrase ‘along the corridor and up the stairs’ as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is ‘x is a cross, wise up!’* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * (Given a grid with the point (6, 1) indicated) Benny describes this point as (1, 6). Jenny describes the point as (6, 1). Who do you agree with? Why?
* Two vertices of a rectangle are (5, 2) and (4, 0). What could the other two vertices be? How many solutions can you find?
* Always / Sometimes / Never: A mirror line touches the shape that is being reflected
* Always / Sometimes / Never: Translations are easier than reflections

NCETM: [Geometry: Position Direction and Movement Reasoning](https://www.ncetm.org.uk/public/files/18436990/9_Progression_Map_Geometry_position_direction_and_movement_Reasoning.pdf) | KM: [Moving house](http://kangaroomaths.com/free_resources/teaching/geometry/moving_house_v2.docx)KM: [Stick on the Maths SSM3: Orientation and reflection of shapes](http://www.kangaroomaths.com/free_resources/teaching/sotm/level3/3ssm3_ewb.doc)NRICH: [Transformations on a Pegboard](http://nrich.maths.org/public/viewer.php?obj_id=1813)NRICH: [Square Corners](http://nrich.maths.org/public/viewer.php?obj_id=1142)NCETM: [Activity A: Translation or Destination](https://www.ncetm.org.uk/resources/42944)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
* When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line.
* Some pupils will confuse the order of x-coordinates and y-coordinates
* When constructing axes, some pupils may not realise the importance of equal divisions on the axes
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| *Presentation of data* | *8 hours* |
| **Key concepts** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * solve comparison, sum and difference problems using information presented in a line graph
 |
| [Return to overview](#Overview) |
| Possible learning intentions | Possible success criteria |
| * Solve problems involving graphs
 | * Understand the difference between a line graph and a bar-line chart
* Identify when a line graph is an appropriate way to show data
* Read values from a line graph
* Answer one-step questions about data in line graphs (e.g. ‘How much?’)
* Answer two-step questions about data in line graphs (e.g. ‘How much more?’)
* Solve problems using information presented in a line graph
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Interpret and construct a simple bar chart
 | DataScaleAxisGraphFrequencyTime graph, Time seriesLine graphBar-line graph, vertical line chartMaximum, minimum | William Playfair, a Scottish engineer and economist, introduced the line graph in 1786.Note: Stage 5 focuses on solving problems using information presented in a line graph. Pupils construct simple line (time) graphs in Stage 4.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils always check they understand the scales used on the axes before attempting to solve problems.*  |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a line graph and tell me a story about it. And another. And another.
* What is the same and what is different: Bar chart, bar-line chart, time graph, line graph?
* Convince me that a line graph is not the same as a bar-line graph.

NCETM: [Statistics Reasoning](https://www.ncetm.org.uk/public/files/18437062/10_Progression_Map_Statistics_Reasoning.pdf) | KM: [Stick on the Maths HD4: Frequency diagrams and line graphs](http://www.kangaroomaths.com/free_resources/teaching/sotm/level4/4hd4_ewb.doc)KM: [Stick on the Maths HD7: Line graphs](http://www.kangaroomaths.com/free_resources/teaching/sotm/level5/5hd7_ewb.doc)NRICH: [Take Your Dog for a Walk](http://nrich.maths.org/4803)NCETM: [The Mathematics of Mountains](https://www.ncetm.org.uk/resources/28590) **Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that a line graph is appropriate for discrete data
* Some pupils may think that a line graph is the same a bar-line chart
* Some pupils may think that one centimetre represents one unit.
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