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Mark one point on the circumference of the first circle, two points on the circumference of the second, and so on. On each circle, join every point to every other point. How many regions?\*

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| --- | --- |
| Prove that x + y = 90° in this right angled triangle**x****y** | Prove that A + B = 180° in this diagram**A****B** |
| Prove that the opposite angles in a rhombus are equal | Prove that the sum of the exterior angles of a quadrilateral is 360°**P****Q****R****S** |
| Prove that the sum of the interior angles of a hexagon is 720° | Trade for a first set of hints |
| Trade for a second set of hints |
| Trade for a third set of hints |

These hint cards can be rearranged to provide a proof, but diagrams will need to be constructed to accompany some steps

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| --- | --- |
| The angles in a triangle sum to 180° | Let the right angle be ‘z’ |
| Therefore x + y = 90° | So x + y + z = 180° |

|  |  |
| --- | --- |
| so A + C = 180° | Angles on a straight line add to 180° |
| Corresponding angles are equal |
| so B = C | Therefore A + B = 180° |

|  |  |
| --- | --- |
| Alternate angles are equal | Add a diagonal to the rhombus |
| and G = M | Therefore F + G = L + M |
| the second pair of opposite angles are equal | so one pair of opposite angles in a rhombus are equal |
| so F = L | The second diagonal of the rhombus shows that |

|  |  |
| --- | --- |
| is the same as the sum of the interior angles in the hexagon | The sum of the interior angles in these triangles |
| This splits the hexagon into four triangles | Therefore the angle sum of the hexagon is 720° |
| Draw all the diagonals from point A | Four triangles have an angle sum of 4 × 180° = 720° |

\* Although the sequence starts 1, 2, 4, 8, 16, … it does not continue as powers of 2.

In fact, the next number in the sequence can be either 30 or 31, depending on whether the points are equally spaced or not. If the points are not spaced equally around the circumference of the circle, then the number of regions can be found using the formula:

$$\frac{1}{24}\left(n^{4}-6n^{3}+23n^{2}-18n+24\right)$$

Substitute *n* = 1, *n* = 2, ... into this formula.