# Goldbach’s theorem

There are lots of odd and unexplained facts about prime numbers. Christian Goldbach (1690 – 1764) was a German mathematician and historian who made some interesting discoveries. He suggested in a letter to a friend that every even integer greater than 2 could be written as the sum of two prime numbers, like this for example:

12 = 5 + 7 20 = 7 + 13

Unfortunately, while mathematicians believe it to be true, no-one has ever been able to prove it. Therefore this remains a 'Goldbach Conjecture' rather than a 'Goldbach Theorem'. If you managed to, you would probably become famous.

* Find pairs of prime numbers that sum to each of the even integers between 30 and 50.

An alternative conjecture is that ‘any number is either prime or is the sum of two primes’ – including odd numbers as well as even numbers.

* Ignore the number 1. How many odd numbers less than 100 can you find that disprove this conjecture?

Corollary: any number is the sum of at most three primes.

# x2 + x + 41

Does this expression always generate a prime number? For example:

12 + 1 + 41 = 43

22 + 2 + 41 = 47

What if we put other prime numbers in this expression?

Do these expressions generate prime numbers? For example:

x2 + x + 13

x2 + x + 23

# Numbers of factors

Prime numbers have two factors – themselves and 1.

* What types of numbers have three factors?
* What types of numbers have four factors?
* What types of numbers have five factors?
* What about other numbers of factors?

# n² and n² + n

Is there always a prime number between n² and n² + n (for n>1)?

# Growing squares

Complete and extend the table below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | 31 |
| 17 |  |  |  |  |  |
| 10 | 11 | 12 | 13 | 22 |  |
| 5 | 6 | 7 | 14 |  |  |
| 2 | 3 | 8 | 15 |  |  |
| 1 | 4 | 9 | 16 |  |  |

* Highlight the primes.
* Do any patterns emerge?
* Use multilink cubes to show the prime numbers (or any other numbers). How does the pattern below represent the numbers above? What other squares should be yellow? Why is the bottom row all shaded the same?

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

* What other number patterns can you see? Can you explain them?
* What if you arranged the numbers in another way – e.g. in a spiral?

# Factors of square numbers; prime factors

When square numbers are expressed as a product of their primes, we may see some connection with the number of factors.

1 4 9 16 25 ……..

16 = 2 × 2 × 2 × 2 = 24. 16 has 5 factors – 1, 2, 4, 8 and 16.

64 = 2 × 2 × 2 × 2 × 2 × 2 = 26. 64 has 7 factors.

* Does 32 have 3 factors?
* Does 34 have 5 factors?
* Is there a connection?
* Does the rule only work for square numbers?

# Artistic Eratosthenes sieve

Using multilink cubes, model the sieve by making a 100 square using different colours for different numbers of factors. A spiral could also be constructed.

Is it possible to make a cubic sieve? e.g. how would you organise the first 8 numbers? 27 numbers?

# The Mersenne primes

In the 17th century a French monk called Father Mersenne studied numbers using the rule 2n – 1, where n is a prime number. For example:

22 – 1 = 3

23 – 1 = 7

Father Mersenne predicted that the following values for n would all yield primes – 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257. He was wrong about a couple of these. What happens for 211-1?

# Lowest common multiples of sequences of whole numbers plus 1

LCM {1, 2} = 2 2 + 1 = 3

LCM {1, 2, 3} = 6 6 + 1 = 7

LCM {1, 2, 3, 4} = 12 12 + 1 = 13

LCM {1, 2, 3, 4, 5} =

How far does this sequence work? Why does it break down?

# n² and (n + 1)²

Is there always a prime number between n² and (n + 1)²? For example:

3² = 9 and (3 + 1)² = 4² = 16: Between 9 and 16 there are two primes – 11 and 13.

* Explore for other numbers

**n² + 1**

Does n² + 1 produce an infinite number of primes?

**n! + 1 and n! – 1**

3! = 3 × 2 × 1 = 6

So 3! + 1 = 6 + 1 = 7, and 7 is prime.

* Is 4! + 1 prime? What about 4! - 1?