Rich Tasks

Plenty is said about 'rich tasks' in mathematics these days. So what exactly is a rich task, and is there anything new about it? Well, nrich have set themselves up as the advocators of all things rich in mathematics, so where better to start if we are looking for a definition:

> *Rich tasks open up mathematics. They transform the subject from a collection of memorised procedures and facts into a living, connected whole. Rich tasks allow the learner to 'get inside' the mathematics. The resulting learning process is far more interesting, engaging and powerful; it is also far more likely to lead to a lasting assimilation of the material for use in both further mathematical study and the wider context of applications.*
>
> *Rich tasks can enable students to work on mathematics in some of the following ways:*
>
> - *Step into activities even when the route to a solution is initially unclear*
> - *Getting started and exploring is made accessible to pupils of wide ranging abilities*
> - *Pose as well as solve problems, make conjectures*
> - *Work at a range of levels*
> - *Extend knowledge or apply knowledge in new contexts*
> - *Allow for different methods*
> - *Have opportunities to broaden their problem-solving skills*
> - *Deepen and broaden mathematical content knowledge*
> - *Have potential to reveal underlying principles or make connections between areas of mathematics*
> - *Include intriguing contexts*
> - *Have opportunities to observe other people being mathematical or see the role of mathematics within cultural settings*
>
> *www.nrich.maths.org*

A more concise definition from the National Strategies is that rich tasks:

- *are accessible and extendable*
- *allow learners to make decisions*
- *involve learners in testing, proving, explaining, reflecting and interpreting*
- *promote discussion and communication* (see 'Establishing Ground Rules')
- *encourage originality and invention*
- *encourage 'what if' and 'what if not' questions*
- *are enjoyable and contain the opportunity for surprise*

Developing a Scheme of Work, SNS, 2007

Evidently, these characteristics of mathematical work have existed for quite some time. However, the implications of the new Programmes of Study provide an added impetus to the development of such work in mathematics. It is a challenge to provide all of these characteristics in one piece of work (or 'task') all of the time, but they can be encouraged through carefully chosen and thoughtfully designed activities.

Conversely, there are some tasks which are traditionally viewed as rich which are in fact quite the opposite due a lack of care or thought in the design. Taking the popular resource, 'Tarsia' as an example, consider the following few questions and answers that could form part of a matching activity that involves simplifying fractions:

$\frac{5}{10}$	$\frac{1}{2}$
$\frac{18}{20}$	$\frac{9}{10}$
$\frac{14}{35}$	$\frac{2}{5}$
$\frac{30}{45}$	$\frac{2}{3}$
$\frac{42}{96}$	$\frac{7}{16}$

This simplistic activity is not really very different from a traditional textbook exercise, and in fact is probably easier to complete as the answers are available (although this self-checking feature does of course have its place). However, the biggest problem is that a learner could work from the simplified fraction and find an equivalent fraction to match – arguably an easier process - so there is no guarantee that the desired learning outcome will be achieved.

To exemplify the desired aims of an effective activity we will consider two resource types, first the APP-based resource; 'Stick on the Maths', and we will then discuss how 'Tarsia' can be used in a rich sense.

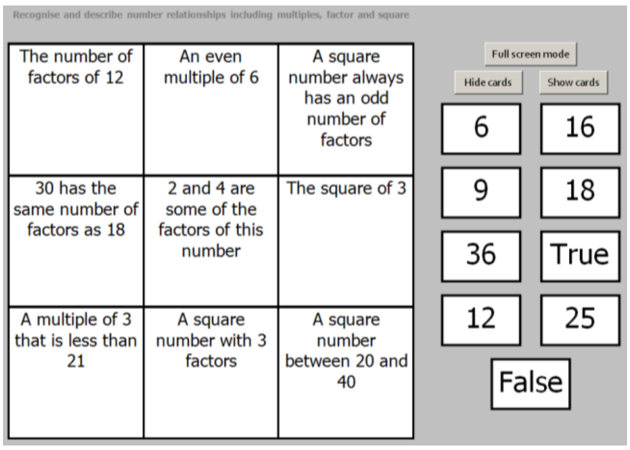
Stick on the Maths

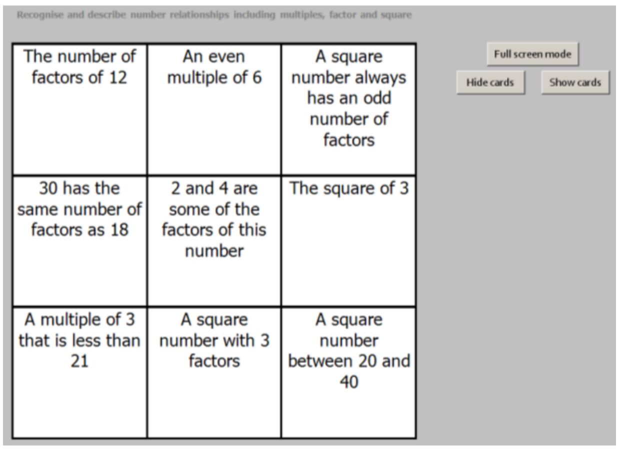
Summary
Stick on the Maths is an engaging and flexible teaching and learning resource which provides a separate activity for each of the APP assessment criteria within ‘Numbers and the Number System’, ‘Calculating’, ‘Algebra’, ‘Shape, Space and Measures’ and ‘Handling Data’. There are no specific activities written for the ‘Using and Applying’ assessment criteria since these judgements require a more general observation of pupil behaviour. However, such judgements could be made throughout all the activities due to their rich nature as described below.

Design
A Stick on the Maths activity comprises a 3 by 3 grid with nine questions, statements or scenarios. Alongside this there are nine cards which can be placed on to the grid to provide a solution or fact that matches the question (statement or scenario).

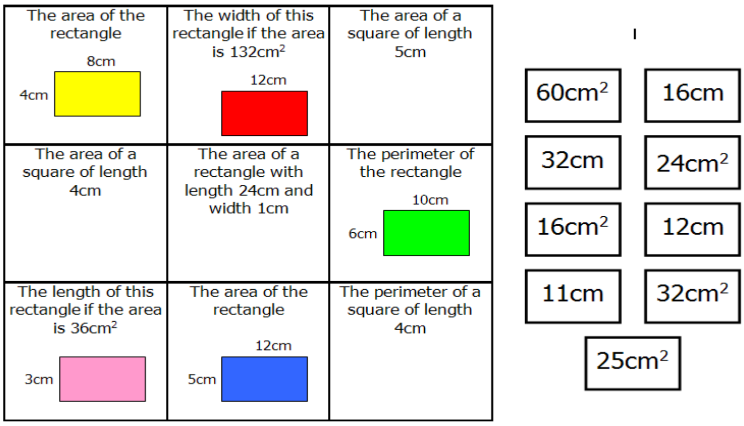
Each activity is provided as a paper version (‘ws’) and an electronic whiteboard version (‘ewb’). The paper version can be used, particularly with the cards cut out, to enable pupils to approach the activity kinaesthetically. The electronic version allows you to view the activity ‘full screen’, and move the ‘solution’ cards by clicking and dragging the border of the box. The two versions can be used together: ‘at the board, on the desk, in the head’! To use the electronic whiteboard version you will need to have macros enabled within Microsoft Word.

Within the electronic version you can also choose to hide the ‘solution’ cards for an alternative way of using the resource. Many of the activities can be used without the cards by challenging learners to produce their own set of solutions. These could be could be shared with a partner, or written on post-it notes and placed on the projected image (hence, ‘Stick on the Maths’). There will then be the opportunity to pick out particular solutions that are of interest and worthy of justification.



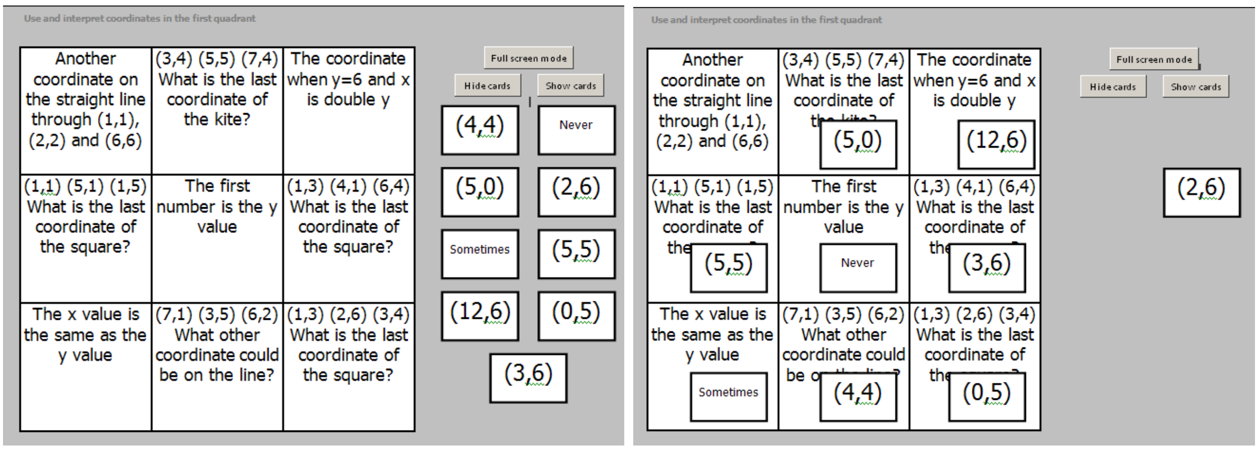


One of the keys to the rich nature of the activities is that the cards are chosen so that common misconceptions are included. These will be matches for other cells on the grid, but the misconceptions may arise through the dialogue surrounding the piece of work. This should be drawn out by the teacher and used to further learning. As an example consider the following activity (L5SSM7):



Two types of misconception in particular are included here. Firstly, if a pupil believes that you find area by adding up the side lengths they might want to place the '24cm²' card on the top left cell. Likewise this confusion between area and perimeter might result in '60cm²' being placed on the middle right cell. Also, if a pupil is not clear about the appropriate unit, the fact that '16cm' and '16cm²' are both included should provide opportunity to address this.

Another feature of Stick on the Maths which contributes to its rich nature is the fact that many of the cards can be placed in more than one position on the grid. As a result, it might be possible for many of the cards to be placed correctly, only to find that the last one or two cannot be placed. This conflict is likely to result in some interesting reasoning, and again the surrounding dialogue is important. Consider the following example (L5ALG2):



Dispelling the myths

- Stick on the Maths is not designed to be used as a test
- A pupil does not necessarily need to place all nine cards correctly in order for you to feel confident that they are secure with a certain assessment criteria. Above all, the observation of the pupil behaviour/response is important.
- Stick on the Maths is not designed to be the only piece of evidence needed to demonstrate security with a certain assessment criteria. This is certainly the case with some of the Handling Data activities.

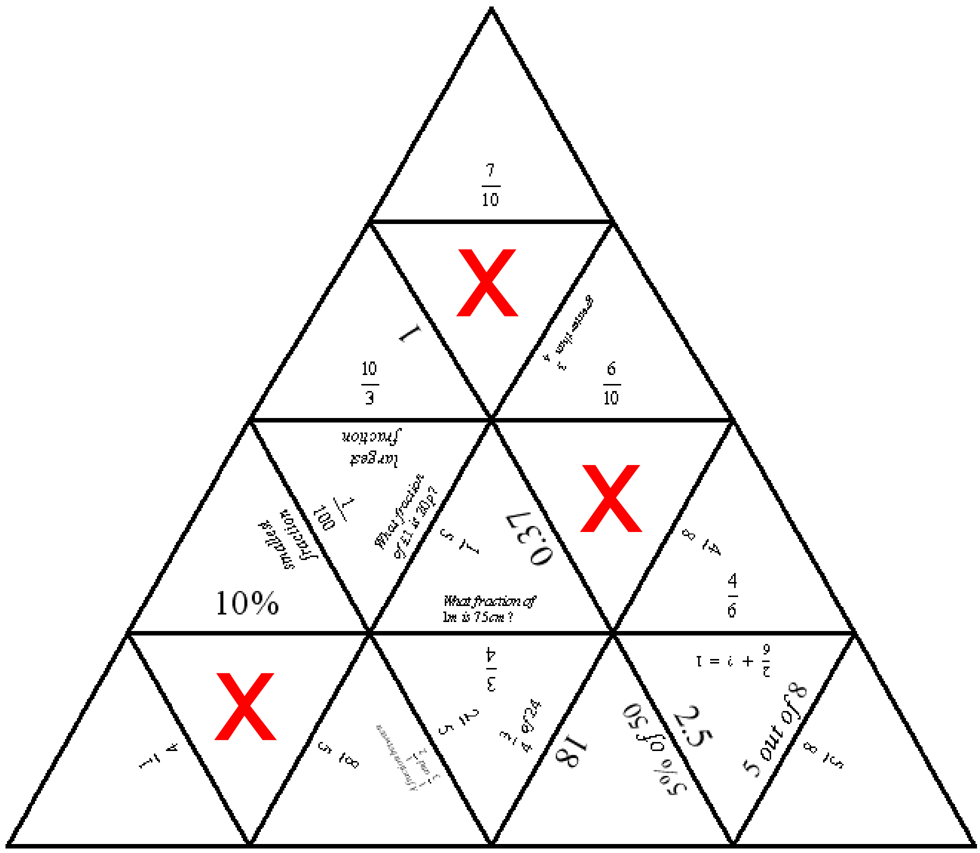
Tarsia

Summary
Tarsia is the name given to a type of freely available jigsaw software and the activities it produces. There are a multitude of designs available, including jigsaws, dominoes and loop cards. They all work on the premise of matching two corresponding entries; for example equivalent statements, or a question and answer.

As described above, the way in which a Tarsia activity is written can dramatically affect its richness. There are many of these activities available on the internet so it is worth making considerations before choosing to use them. The following guidance is split into using ready-made activities' and 'designing your own'.

Using ready-made activities
Any of the following three ideas can be used as independent, paired or group work. It is essential that the teacher follows up any errors and misconceptions that are experienced at each stage and for each option.

1. Using the 'solution' tab, provide the 'whole' with some pieces missing – indicated on the following diagrams by pieces marked with an 'x':



Then:

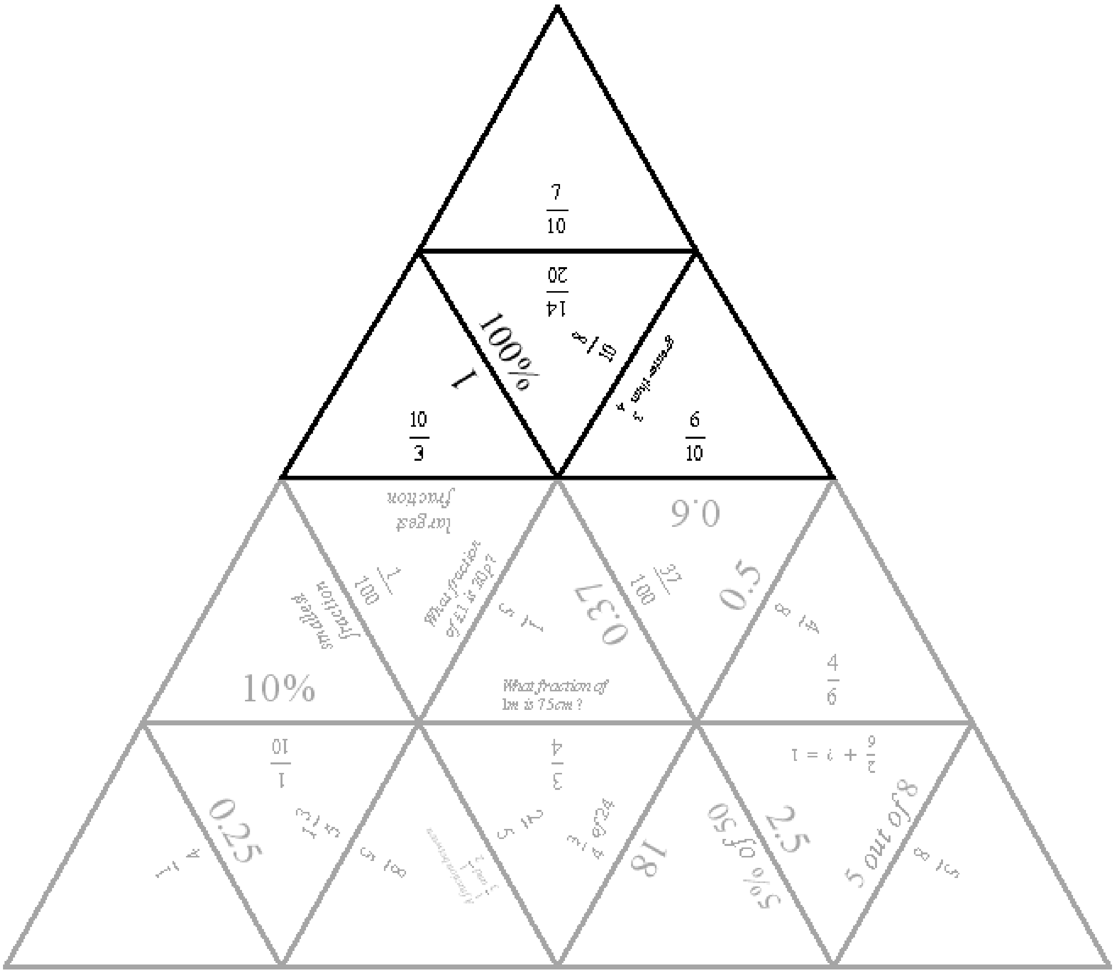
a) Ask learners to justify the solutions that have been matched; e.g. 'Convince me that ¾ of 24 is 18'.

b) Ask learners to provide solutions that would complete the jigsaw, or

c) Provide the three missing pieces and ask learners to place them correctly

2. Ask learners to use the jigsaw pieces to find as many ‘solution pairs’ as they can

3. Crop the ‘solution’ page to provide a smaller, more accessible, subset of pieces. For example, break down the solution into four subsets and ask each group member to complete a subset and then work together to solve the whole.



A 16-piece domino activity can broken down into 4 groups of four pieces too.

NB: ALWAYS TRY IT YOURSELF FIRST!

This is important to:
a) quality assure the activity for its ‘richness’ (see fundamentals on next page),
b) make sure it is ‘fit for purpose’,
c) identify potential hazards, challenges and misconceptions.

Be prepared to:
a) adopt it (as it stands)
b) adapt it (to meet needs)
c) innovate (create your own) – read on for more information

Creating your own Tarsia activities

The ‘fundamentals’ for a rich Tarsia activity are:

- The purpose of the activity is clear
- A balance of ‘unique’ and ‘multiple solution’ question or problems (see coloured examples below). **Be aware that too many ‘multiple solution’ questions make the task very difficult even if the mathematical content is quite straightforward**. A limit of two ‘multiple solution’ questions is often manageable, and it pays to try out the resulting activity first.
- The use of negative statements (see italicised example below)
- A range of vocabulary
- Opportunities to address errors and misconceptions are exploited (see underlined example below – this is based on a muddled understanding of factors and multiples; some learners may think that 48 is the smallest factor of 24)
- Opportunities for mathematical reasoning are exploited, possibly through the use of ‘True / False’ **or** ‘Sometimes, Always, Never’ (see emboldened example below).

2 is the largest remainder when the divisor is 3	**True**
Odd factors of 14	7 and 1
Quotient of 36 ÷ 4	9
A calculation equivalent to 40 ÷ 5	4 × 2
Balances 18 + 22	4 × 10
Not a multiple of 2	11
Remainder of 4 if divided by 5	14
Quotient = 2.5	25 ÷ 10

The fourth multiple of 12	<u>48</u>
<u>Smallest factor of 24</u>	?
Divide 60 by 3	20
Remainder of 1	19 ÷ 6
3 × 3 = ___ ÷ ___	18 and 2
Quotient of 5	35 ÷ 7
Even multiple of 5	10
All multiples of 2 are multiples of 4	**False**

Other options include:

- Start with a smaller number of pieces design
- Handwrite the activity first
- Leave a ‘?’ for learners to supply the solution and demonstrate completion of the task (as above)

Dispelling the Myths

- Tarsia is not necessarily a rich task! It all depends how it is designed and used.
- The opportunities to address 'Using and Applying' skills should not be overlooked – a well designed Tarsia will result in a great deal of problem-solving and reasoning.
- You do not have to share the outline shape of the puzzle that learners are aiming for – withholding it makes it a substantially more difficult task. If approaching the task in this way, providing the shape at some point is an incredibly useful hint.
- It might sometimes be appropriate to include a larger proportion of 'multiple solution' questions. A Tarsia designed in this way, but using straightforward mathematical content could be appropriate for able learners:

Establishing Ground Rules

The characteristics of rich tasks as discussed on page 1 make it clear that collaborative work and dialogue will feature within this type of work. This does not come without an additional challenge however, and learners need to know ground rules and expectations in terms of how to approach this type of work. For example:

- Share ideas and listen to each other
- Respect each other's opinions
- Enjoy mistakes
- Don't be a passenger

For further guidance on this, the following publications could prove very useful:

- Improving Learning in Mathematics - Discussing Maths, DfES, 2005
- Fostering and Managing Collaborative Work, Bowland Maths